Packets Travelling in Non-homogeneous Networks

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ABSTRACT

This paper considers a probability model for travel of a packet from a source node to a destination node in a large non-homogeneous multiple hop network with unreliable routing tables. Use of a random model is justified by the lack of precise information that can be used in each step of the packet's travel, and randomness can also be useful in exploring alternate paths when a long sequence of hops has not resulted in the packet's arrival to the destination. The packet's travel may also be impeded if certain routers on its path prove to be unreliable, or the packet may be dropped from a buffer or destroyed due to packet loss. The packet also has a limited time-out that allows the source to retransmit a dropped or lost packet. Because the network itself may be extremely large, we consider packet travel in an infinite random non-homogeneous medium, with events that may interrupt, destroy or stop the packet from moving towards its destination. We derive a numerical-analytical solution allowing us to compute the average travel time of the packet from source to destination, as well as to estimate its energy consumption. Two interesting applications are then presented. In the first one a wireless network where areas which are remote from the source and destination nodes may have poor wireless coverage so that the packet losses become more frequent as the packet "unknowingly" (due to poor routing tables for instance) meanders away from the source and destination node. The second application is related to defending a destination node against attacks that take the form of packets that carry a virus or a worm that can be detected via deep packet inspection at intermediate nodes, and as the packet approaches the destination node it is more frequently inspected and dropped if it is a threat.

Categories and Subject Descriptors

C.2 [**Computer Communication Networks**]: Network Architecture and Design—*Distributed networks*
; C.4 [**Performance of Systems**]: Modeling techniques; G.3 [**Mathematics of Computing**]: Probability and Statistics—*Markov processes*

General Terms

Performance, Reliability, Theory

Keywords

Brownian motion, diffusion process, multi-hop networks

1. INTRODUCTION

Search for recognisable objects in large random or imprecisely known environments has been studied in robotics, biology, physics, transportation and communication networks [12, 22, 21, 18, 19]. This paper focuses on the travel of a packet from a given source to a destination which is at distance D from the packet, but whose whereabouts are unknown, or imprecisely known. Thus the packet may not have precise information about which direction it should pursue as it moves from one hop to the next [14, 8, 10], and we suppose that the destination node is recognised only when the packet gets close to it, typically one hop away. Furthermore, when the network is infinitely large the packet may become lost in remote areas. The packet can also be destroyed by its own finite life-time, by a failure or error in the communication system or by buffer overflow in an intermediate node.

Random walk models for networks were analysed in [20], and in [15] the mean and variance of the hitting time were obtained for a torus-lattice network graph when the next node visited is selected at random among all neighbours, showing that in such networks the probability distribution of packet delivery time is approximately geometrically distributed. For multiple independent unbiased random walks on a connected network [23] it was shown that the mean first passage time converges to the shortest path between the source and the destination as the number of travellers approaches infinity. A random walk over a wireless network with uniform node distribution and circular symmetry was evaluated in [1]. In [4], the probability of the unbiased traveller visiting a particular node in a given step was derived for a 2-D grid topology, but unbiased routing was shown to achieve poor performance [2] because the random walker may "orbit" around the target node for a long time before attaining it.

In [9] it was shown that the time it takes a packet to travel from a source node to a destination node in an infinitely large and unreliable network is finite on average, provided that a time-out mechanism is inserted to destroy the ongoing packet after a predetermined time, and replace it with a new packet that starts at the same source and proceeds at random and independently of its predecessor. Since the network is infinite, the time-out also protects the packet from spending an unreasonably long time in remote areas from which it may never return. By using a randomly different travel path, the new packet takes a distinct path from its previous incarnation, increasing its chances of reaching the destination.

The use of Brownian motion approximations simplifies the analysis for very large networks, and the time dependent solution for the passage time using Brownian motion was considered in [3] where, differently from our work, losses and retransmissions occur at specific distances from the destination. In [11] N packets are simultaneously, but independently, sent out in the quest for the same destination node. Both the total travel time and the energy expended were obtained using a Brownian motion model of the travel process.

While most previous work has dwelt on a spatially homogeneous network, here we will consider a packet travelling in a *non-homogeneous* multi-hop network. One motivation for this work is the case where some parts of the network may be particularly faulty or degraded while the rest of the network is operating properly. Thus the network's operational quality may be quite good close to the source node, but it may become less reliable when the packet moves far away from it. This paper is motivated by two interesting applications. The first case relates to a wireless network where remote areas, away from where the source and destination nodes are located, perhaps have poor wireless coverage so that the packet losses become more frequent as the packet "unknowingly" (due to poor routing tables for instance) meanders away from the source and destination node. The second one is related to defending a destination node against attacks that take the form of packets that may carry a virus or a worm that can be detected via deep packet inspection at some intermediate nodes. Thus as a packet approaches the destination node it may be inspected by intermediate protecting nodes and dropped if it is viewed as a threat; however the source of the packet will use a time-out to attempt sending the attacking packet forward again and it is interesting to see whether the attacker will eventually be successful. Another example of non-homogeneous wireless network occurs when the packet progresses faster as it approaches the destination, for instance when directional information such as a radio signature becomes stronger as the packet approaches the destination node. However we will not examine this case in detail in this paper.

In the sequel we will model a packet's motion towards a destination node in an infinite random non-homogeneous network, with packet drops that will stop the packet's progress resulting in a subsequent time-out retransmission of the packet from the source. We obtain an exact expression for the average time and energy that it takes the packet to eventually find the destination node, based on a non-homogeneous Brownian motion model enhanced with some useful point processes representing the relaunch of an aborted or interrupted search. We develop an analytical solution technique based on a finite but unbounded number of internally homogeneous segments, yielding the average travel time and the energy expended. Then the results are applied to the two cases of interest that we have outlined.

2. AN ANALYTICAL MODEL FOR NON-HOMOGENEOUS PACKET TRAVEL

Following the approach in [11] let Y_t be the packet's distance from the destination node at time $t \geq 0$. Clearly the packet starts at distance $Y_0 = D$ and the travel process ends at some time T defined by:

$$T = \inf\{t : Y_t = 0\}$$

The random variable $s(t)$ represents the state of the packet at time $t \geq 0$ where $s(t) \in \{\mathbf{S}, \mathbf{W}, \mathbf{P}, \ ...\}$, and $s(t)$ is in:

- **S**: if the travel is proceeding and the packet's distance from the destination is $Y_t > 0$. The probability density function of the position Y_t is represented by $f(z,t)dz = P[z < Y_t \leq z + dz, \ s(t) = \mathbf{S}]$.

- **W**: its life-span has ended, and so has its travel. This can happen because the packet was destroyed or became lost, and the source was informed via the time-out. After an additional exponentially distributed delay of parameter μ, meant to avoid mistakes in assuming that the packet is lost, a new packet is placed at the source and a new travel immediately begins. We write $W(t) = P[s(t) = \mathbf{W}]$.

- **L**: if the packet is destroyed or lost, and the travel is interrupted until a new packet can be sent out. The time spent in this state is exponentially distributed with parameter r which is the same parameter as that of the "time-out" or life-span, since the source realises that the packet is lost or destroyed via the life-span or time-out effect. At the end of this exponentially distributed time, the packet is handled just as if it has been lost, and we denote $L(t) = P[s(t) = \mathbf{L}]$.

- **P**: the packet has reached its destination and the travel process ends. However, as an artefact to construct an indefinitely repeating recurrent process, after one time unit the travel process restarts at the source and a new packet is sent out. We will use the notation $P(t) = P[s(t) = \mathbf{P}]$.

Notice that the above process repeats itself indefinitely, and $E[T]$ is the average time that it takes from any successive start of the travel until the first instance when state **P** is reached again. Let $P(t)$ be the probability that the model we have just described is in state **P** at time $t \geq 0$, and let $P = \lim_{t\to\infty} P(t)$. Then:

$$P = \frac{1}{1 + E[T]}, \quad E[T] = P^{-1} - 1$$

During the packet's travel in state **S** while $\{Y_t = z > 0\}$ the following events can occur in the time interval $[t, t + \Delta t[$:

- With probability $\lambda(z)\Delta t + o(\Delta t)$ the packet is destroyed or lost, and enters state **L**. From that state it enters state **W** after an exponentially distributed delay of parameter r.

- With probability $r\Delta t + o(\Delta t)$ the packet's life-span runs out and it enters state **W**. Note that $1/r$ is the average life-span. As indicated earlier, when it enters state $\mathbf{W}$, after an additional delay of average value $1/\mu$, the packet is replaced with a new one at the source.

The average rate per unit time at which the packet approaches the destination node when it is at distance z is denoted by $b(z)$, and the variance of the distance travelled in the interval $[t, t+\Delta t[$ is denoted by $c(z)\Delta t$ so that:

$$b(z) = \lim_{\Delta t \to 0} \frac{E[Y_{t+\Delta t} - Y_t | Y_t = z]}{\Delta t},$$
$$c(z) = \lim_{\Delta t \to 0} \frac{E[(Y_{t+\Delta t} - Y_t)^2 - (E[Y_{t+\Delta t} - Y_t])^2 | Y_t = z]}{\Delta t}$$

When $b(z) < 0$, on average the packet gets closer over time to the destination node, but $b(z) \geq 0$ is also possible. Note that P is a fictitious state that we use to create a recurrent random process which indefinitely repeats itself. The packet's distance to destination in a non-homogeneous medium is represented by the probability density function $f(z,t)$ representing the position of the packet at time $t \geq 0$, and assume that it satisfies the position dependent diffusion equation [5, 16]. Such models have been used previously to represent traffic in communications systems and transportation systems [17, 6, 7, 13].

The model for the non-homogeneous medium is simplified to a finite but unbounded number of "segments" that have different parameters for the Brownian motion describing the packet's movement as a function of its distance to the destination node, while within each segment the parameters are the same. The first segment is in the immediate proximity of the destination node, starting at distance $z = 0$. Each segment may have a different size, and we assume that there are a total of $m < \infty$ segments. By choosing as many segments as we wish, and letting each segment be as small as we wish (all segments need not be of the same length), we can approximate as closely as needed any physical situation that arises where the packet's motion characteristics vary over the distance of the packet to the destination node. If the point $0 \leq Z_k < \infty$ is the boundary between the k-th and $(k+1)$-th segments with $Z_0 = 0$, with the last segment going from Z_{m-1} to $+\infty$, with m and Z_{m-1} being finite but unbounded, we can take as many segments as needed, and they are all finite except the last segment. Thus for $1 \leq k \leq m$, the k-th segment represents the range of distances $Z_{k-1} \leq z < Z_k$; let $S_k = Z_k - Z_{k-1}$ denote its size and write $\{f(z,t), b(z), c(z), \lambda(z)\} = \{f_k(z,t), b_k, c_k, \lambda_k\}$. We use n to denote the segment number in which the source node is located, i.e. $Z_{n-1} < D \leq Z_n$. The differential equation for the *stationary* solution of the location dependent diffusion equation for segment $k \neq n$ is then:

$$0 = \frac{c_k}{2}\frac{d^2 f_k(z)}{dz^2} - b_k \frac{df_k(z)}{dz} - (\lambda_k + r) f_k(z) \quad (1)$$

while the equation for the segment where the source is located is:

$$-[P + \mu W]\delta(z - D) = \frac{c_n}{2}\frac{d^2 f_n(z)}{dz^2} - b_n \frac{df_n(z)}{dz} - (\lambda_n + r) f_n(z) \quad (2)$$

We also have:

$$rL = \sum_{k=1}^{m} \lambda_k \int_{Z_{k-1}}^{Z_k} f_k(z)dz \quad (3)$$

$$\mu W = r[L + \sum_{k=1}^{m} \int_{Z_{k-1}}^{Z_k} f_k(z)dz] \quad (4)$$

$$P = \lim_{z \to 0^+} [\frac{c_1}{2}\frac{df_1(z)}{dz} - b_1 f_1(z)] \quad (5)$$

and the normalisation condition:

$$1 = P + W + L + \sum_{k=1}^{m} \int_{Z_{k-1}}^{Z_k} f_k(z)dz \quad (6)$$

Result 1 Let u_k, v_k be, respectively, the positive and negative real roots of the characteristic polynomial of the stationary equation for the k-th segment:

$$u_k, v_k = \frac{b_k \pm \sqrt{b_k^2 + 2c_k(\lambda_k + r)}}{c_k}$$

Then the total average travel time, which is obtained by solving for P so that $E[T] = P^{-1} - 1$, is given by:

$$E[T] = (\frac{1}{r} + \frac{1}{\mu}) \times$$
$$[\sqrt{\frac{b_n^2 + 2c_n(\lambda_n + r)}{b_1^2 + 2c_1(\lambda_1 + r)}} \frac{\overline{A}_n \overline{G}_n e^{u_n S_n} - \overline{B}_n \overline{F}_n e^{v_n S_n}}{\overline{G}_n e^{u_n(Z_n - D)} + \overline{F}_n e^{v_n(Z_n - D)}} - 1] \quad (7)$$

where the remaining parameters are computed as follows. Define:

$$\alpha_k^- = \frac{c_k u_k - c_{k-1}v_{k-1}}{c_k(u_k - v_k)}, \quad \beta_k^- = \frac{c_k u_k - c_{k-1}u_{k-1}}{c_k(u_k - v_k)}$$
$$\alpha_k^+ = \frac{c_k u_k - c_{k+1}v_{k+1}}{c_k(u_k - v_k)}, \quad \beta_k^+ = \frac{c_k u_k - c_{k+1}u_{k+1}}{c_k(u_k - v_k)} \quad (8)$$

Then set $\overline{A}_1 = 1$ and $\overline{B}_1 = -1$ and for $2 \leq k \leq n$ compute:

$$\begin{bmatrix} \overline{A}_k \\ \overline{B}_k \end{bmatrix} =$$
$$\begin{bmatrix} \alpha_k^- & \beta_k^- \\ 1-\alpha_k^- & 1-\beta_k^- \end{bmatrix} \begin{bmatrix} e^{u_{k-1}S_{k-1}} & 0 \\ 0 & e^{v_{k-1}S_{k-1}} \end{bmatrix} \begin{bmatrix} \overline{A}_{k-1} \\ \overline{B}_{k-1} \end{bmatrix} \quad (9)$$

Then set $\overline{F}_m = 0$ and $\overline{G}_m = e^{v_m Z_m}$, and start another computation at $k = m-1$ for $n \leq k \leq m-1$ with:

$$\begin{bmatrix} \overline{F}_k \\ \overline{G}_k \end{bmatrix} =$$
$$\begin{bmatrix} \alpha_k^+ & \beta_k^+ \\ 1-\alpha_k^+ & 1-\beta_k^+ \end{bmatrix} \begin{bmatrix} e^{-u_{k+1}S_{k+1}} & 0 \\ 0 & e^{-v_{k+1}S_{k+1}} \end{bmatrix} \begin{bmatrix} \overline{F}_{k+1} \\ \overline{G}_{k+1} \end{bmatrix} \quad (10)$$

Proof The general solution has the form:

$$f_k(z) = \begin{cases} A_k e^{u_k z} + B_k e^{v_k z}, & Z_{k-1} \leq z \leq \min(D, Z_k) \\ F_k e^{u_k z} + G_k e^{v_k z}, & \max(D, Z_{k-1}) \leq z \leq Z_k \end{cases}$$

Thus there are $2m + 2$ constants to be determined from (a) the boundary conditions at 0 and $+\infty$, (b) the continuity condition of the probability density function at D and at the boundaries between segments, and (c) conditions obtained by integrating the defining differential equation around D

and the boundaries between segments. First consider the case $Z_{k-1} \leq z \leq \min(D, Z_k)$; to ensure continuity of the probability density function at $z = Z_{k-1}$ we have:

$$f_k(Z_{k-1}) = f_{k-1}(Z_{k-1}) \tag{11}$$

or equivalently

$$A_k e^{u_k Z_{k-1}} + B_k e^{v_k Z_{k-1}} = A_{k-1} e^{u_{k-1} Z_{k-1}} + B_{k-1} e^{v_{k-1} Z_{k-1}}$$

Furthermore, integrating the differential equation (1) from $z = Z_{k-1} - \epsilon$ to $z = Z_{k-1} + \epsilon$ and taking the limit as ϵ tends to 0 yields:

$$\frac{c_k}{2}\frac{df_k(Z_{k-1})}{dz} - \frac{c_{k-1}}{2}\frac{df_{k-1}(Z_{k-1})}{dz} = [b_k - b_{k-1}] f_k(Z_{k-1}) \tag{12}$$

or

$$A_k u_k e^{u_k Z_{k-1}} + B_k v_k e^{v_k Z_{k-1}} = \frac{2b_k - c_{k-1} v_{k-1}}{c_k} \times A_{k-1} e^{u_{k-1} Z_{k-1}} + \frac{2b_k - c_{k-1} u_{k-1}}{c_k} B_{k-1} e^{v_{k-1} Z_{k-1}}$$

Solving (11) and (12), we can write A_k and B_k in terms of A_{k-1} and B_{k-1} as:

$$\begin{aligned} A_k e^{u_k Z_{k-1}} &= \alpha_k^- A_{k-1} e^{u_{k-1} Z_{k-1}} + \beta_k^- B_{k-1} e^{v_{k-1} Z_{k-1}} \\ B_k e^{v_k Z_{k-1}} &= [1 - \alpha_k^-] A_{k-1} e^{u_{k-1} Z_{k-1}} \\ &\quad + [1 - \beta_k^-] B_{k-1} e^{v_{k-1} Z_{k-1}} \end{aligned}$$

where α_k^- and β_k^- are defined in (8). From the boundary condition $\lim_{z \to 0^+} f_1(z) = 0$ we have $B_1 = -A_1$; therefore the stationary solution of the differential equation for $Z_{k-1} \leq z \leq \min(D, Z_k)$ can be expressed as follows:

$$f_k(z) = A_1[\overline{A}_k e^{u_k(z - Z_{k-1})} + \overline{B}_k e^{v_k(z - Z_{k-1})}] \tag{13}$$

where the constants $\overline{A}_k$ and $\overline{B}_k$ are computed recursively using the matrix multiplication in (9). Next consider a segment k where $z \geq D$, and write the constants F_k and G_k in terms of F_{k+1} and G_{k+1} by solving boundary conditions similar to (11) and (12) at $z = Z_k$:

$$\begin{aligned} F_k e^{u_k Z_k} &= \alpha_k^+ F_{k+1} e^{u_{k+1} Z_k} + \beta_k^+ G_{k+1} e^{v_{k+1} Z_k} \\ G_k e^{v_k Z_k} &= [1 - \alpha_k^+] F_{k+1} e^{u_{k+1} Z_k} + [1 - \beta_k^+] G_{k+1} e^{v_{k+1} Z_k} \end{aligned}$$

Since $f(z)$ is a probability density function we must have $\lim_{z \to \infty} f_m(z) = 0$ which implies that $F_m = 0$, thus the solution for $\max(D, Z_{k-1}) \leq z \leq Z_k$ is given by:

$$f_k(z) = G_m[\overline{F}_k e^{-u_k(Z_k - z)} + \overline{G}_k e^{-v_k(Z_k - z)}] \tag{14}$$

where $\overline{F}_k$ and $\overline{G}_k$ are computed using (10). Note that $\overline{G}_m = e^{v_m Z_m}$ yields the desired solution for the last segment, that is $f_m(z) = G_m \overline{G}_m e^{-v_m(Z_m - z)} = G_m e^{v_m z}$. In order to determine A_1 and G_m, consider the n-th segment and apply the continuity condition of $f_n(z)$ at $z = D$ so that:

$$\begin{aligned} &G_m[\overline{F}_n e^{-u_n(Z_n - D)} + \overline{G}_n e^{-v_n(Z_n - D)}] \\ &\quad = A_1[\overline{A}_n e^{u_n(D - Z_{n-1})} + \overline{B}_n e^{v_n(D - Z_{n-1})}] \end{aligned} \tag{15}$$

Also, integrating the differential equation (2) from $z = D - \epsilon$ to $z = D + \epsilon$ and taking the limit as ϵ tends to 0 yields:

$$\begin{aligned} \frac{2[P + \mu W]}{-c_n} &= G_m[\overline{F}_n u_n e^{-u_n(Z_n - D)} + \overline{G}_n v_n e^{-v_n(Z_n - D)}] \\ &\quad - A_1[\overline{A}_n u_n e^{u_n(D - Z_{n-1})} + \overline{B}_n v_n e^{v_n(D - Z_{n-1})}] \end{aligned} \tag{16}$$

From (5), the probability P is given by:

$$P = \frac{c_1}{2}(u_1 - v_1)A_1 = \sqrt{b_1^2 + 2c_1(\lambda_1 + r)}\; A_1 \tag{17}$$

Substituting (4) into (6) yields:

$$P + \mu W \left(\frac{1}{r} + \frac{1}{\mu}\right) = 1 \tag{18}$$

Now solving the system of linear equations (15)−(18) we can determine A_1 and G_m:

$$\begin{aligned} A_1 &= \eta\left[\overline{G}_n e^{u_n(Z_n - D)} + \overline{F}_n e^{v_n(Z_n - D)}\right] \\ G_m &= \eta\left[\overline{A}_n e^{u_n S_n} e^{v_n(Z_n - D)} + \overline{B}_n e^{v_n S_n} e^{u_n(Z_n - D)}\right] \end{aligned} \tag{19}$$

where

$$\eta = \frac{r\mu/(r+\mu)}{\sqrt{b_n^2 + 2c_n(\lambda_n + r)}} \Big\{ \overline{A}_n \overline{G}_n e^{u_n S_n} - \overline{B}_n \overline{F}_n e^{v_n S_n} - \sigma[\overline{G}_n e^{u_n(Z_n - D)} + \overline{F}_n e^{v_n(Z_n - D)}] \Big\}^{-1},$$

$$\sigma = [1 - \frac{r\mu}{r+\mu}] \sqrt{\frac{b_1^2 + 2c_1(\lambda_1 + r)}{b_n^2 + 2c_n(\lambda_n + r)}}$$

Substituting A_1 in (17) yields P from which the average travel time follows directly. □

Remark 1 With n being the index of the discretisation segment that includes the source node at D, it is interesting to see that $E[T]$ only depends on a set of parameters that are computed for values of $k = 1$, $k = n$, and on two sets of algebraic iterations between $k = 1$ and $k = n$ and $k = m$ down to $k = n$.

Remark 2 When the source node is in the pen-ultimate segment we have $m = n$, and:

$$E[T] = \frac{r+\mu}{r\mu}[\sqrt{\frac{b_n^2 + 2c_n(\lambda_n + r)}{b_1^2 + 2c_1(\lambda_1 + r)}}\overline{A}_n e^{u_n(D - Z_{n-1})} - 1] \tag{20}$$

For a homogeneous medium $m = n = 1$ and:

$$E[T] = (\frac{1}{r} + \frac{1}{\mu})[e^{u_1 D} - 1]$$

as we would expect from [11].

Remark 3 As in [11], if energy is consumed only when a packet is actually being forwarded through the network, while during wait times for retransmissions the packet (which remains stored at the source until final successful delivery) consumes only negligible energy, then the average energy consumption $E[J]$ until the packet reaches its destination is:

$$E[J] = (1 + E[T]) \sum_{k=1}^{m} \int_{Z_{k-1}}^{Z_k} f_k(z)dz \tag{21}$$

2.1 Greater loss in remote areas

An example of practical interest occurs when a packet that moves far away from its initial point and from the destination node, has a greater chance of being lost or destroyed. This could represent a multi-hop wireless network deployed in a very large area; as the packet moves to remote areas

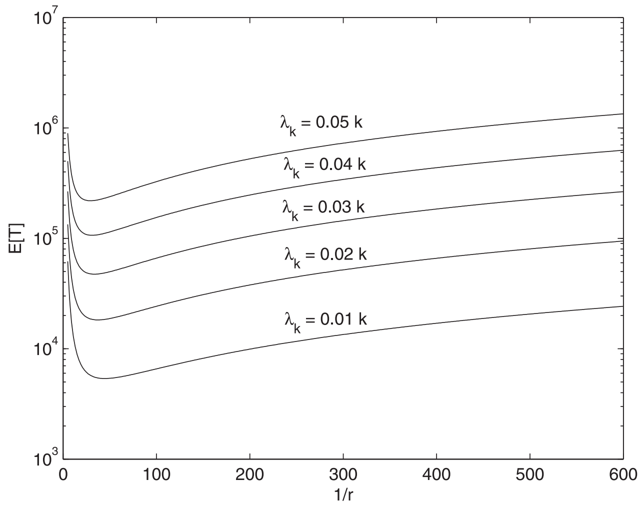

Figure 1: $E[T]$ (logarithmic scale) versus the average time out $1/r$ when loss rates increases linearly and $m = 100$.

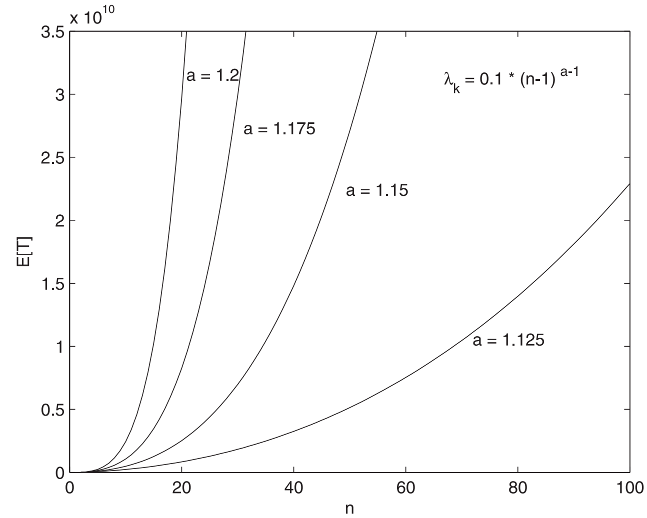

Figure 2: $E[T]$ versus n when $\lambda_k = 0.1(n-1)^{a-1}$ for different values of a; $b_k = -1$, $c_k = 0$, $S_k = D/(n-1)$, $\mu = 0.1$, $r = 0.02$, and $D = 100$.

far from the region where the source and destination are located, the nodes that the packet might visit are less likely to handle it and more likely to just discard it. This can also represent a network where there are fewer nodes in remote areas and inter-node communications in such areas are less reliable. As an example consider 100 segments with $S = 1$ and a loss rate that increases with distance: $\lambda_k = k\ell$, $\ell > 0$, $1 \le k \le 99$, $\lambda_{100} = 100\ell$. If average speed of the packet's motion and its second moment remain constant with $b_k = 0$ and $c_k = 1$ for $1 \le k \le 100$, the results with $D = 10$ in Figure 1 show that a relatively short time-out is needed to optimise the average travel time, but that the resulting optimum is nevertheless very large.

3. RETARDING AN ATTACKING PACKET

Another interesting case arises when the packet that we are modelling contains some form of attack on the destination node, such as a virus or a worm. Also, we suppose that the network protects this particular node by introducing a capability at intermediate nodes to detect the contents of the packet and to drop it. However the sender will then, after a time-out, send the attacking packet again. The question is then whether it is possible to block the attack indefinitely or whether to the contrary the attacking packet will eventually reach the destination node that is being defended.

We examine this problem in the context of a wired network that uses shortest path routing. Thus if the distance D refers to the number of hops from source to destination, and if the routers are operating properly, we will have $b = -1$ and $c = 0$ throughout the network. More generally if there is no uncertainty in routing $c_k = 0$ and $b_k < 0$, and it can be shown that the total average travel time does not depend on the network's parameters for $z > D$:

$$E[T] = \frac{r+\mu}{r\mu}\ [e^{\frac{\lambda_n+r}{|b_n|}D}\ e^{\sum_{k=1}^{n-1}\left(\frac{\lambda_k+r}{|b_k|}-\frac{\lambda_n+r}{|b_n|}\right)S_k} - 1] \qquad (22)$$

Furthermore if the routers are perfect and always provide shortest distance routing we have $b_k = -1$ and:

$$E[T] = \frac{r+\mu}{r\mu}\ [e^{(\lambda_n+r)D}\ e^{\sum_{k=1}^{n-1}(\lambda_k-\lambda_n)S_k} - 1] \qquad (23)$$

Now let us introduce a non-homogeneous packet drop effect by choosing an integer n to create an acceleration in the packet drop effect and let $S_k = D/(n-1)$ so that:

$$E[T] = \frac{r+\mu}{r\mu}\ [e^{(r+\frac{\sum_{k=1}^{n-1}\lambda_k}{n-1})D} - 1] \qquad (24)$$

which yields the following result.

Result 2 If $\lim_{n\to\infty} \frac{\sum_{k=1}^{n-1}\lambda_k}{n-1} = +\infty$ then the packet will never reach the destination node. Otherwise it will reach it in a time which is finite on average, and with probability one.

Figure 2 illustrates Result 2 by showing that even with a small excess, represented by $a > 1$, above the $O(n)$ rate of increase for the loss rate λ_k the attacking packet's progress will be indefinitely impeded by the drops, despite the subsequent time-outs.

3.1 A neighbourhood with traps

As a final and related example suppose that routers in the neighbourhood of the destination node within a distance S contains "traps" that can identify the attacking packet and drop it. Accidental drops of the packet (due to transmission errors or buffer overflows) may also occur at a lower rate. Thus we take $m = n = 2$, so that $E[T]$ is obtained from (20) with $\lambda_2 = \lambda$ and $\lambda_1 = \lambda + \gamma$, $\gamma > 0$:

$$E[T] = \frac{r+\mu}{r\mu}[\sqrt{\frac{b_2^2 + 2c_2(\lambda+r)}{b_1^2 + 2c_1(\lambda+\gamma+r)}}\overline{A}_2 e^{u_2(D-S)} - 1]$$

Figure 3 shows the manner in which $E[T]$ sharply increases with γ, for S ranging between 10 and 15, $D = 100$, $b_2 = b_1 = 0.25$, $c_1 = c_2 = 1$, $\lambda = 0$. Also $\mu = 1/10$ and r is set to the value that minimises $E[T]$ when $\gamma = 0$ and $S = 10$. Figure 4, with $S = 10$ and the same set of parameters, shows that even small increases (more negative) in

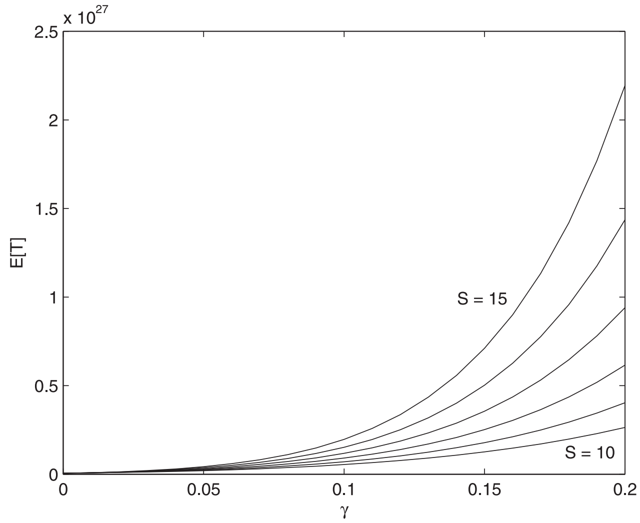

Figure 3: Average travel time $E[T]$ versus $\gamma = \lambda_1 - \lambda_2$ for $S = 10$ to 15 with a step size of 1.

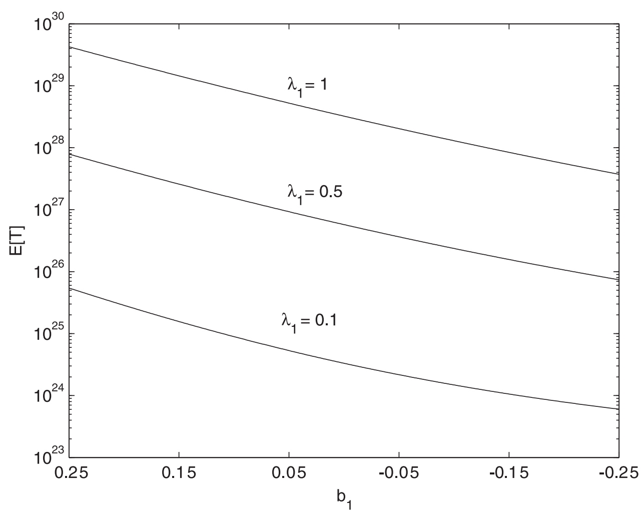

Figure 4: $E[T]$ (logarithmic scale) versus b_1 for different values of λ_1.

average speed at which the packet approaches its objective can reduce average travel time by an order of magnitude, yet $E[T]$ is still very large.

Figures 5 and 6 question how S and λ_1 may be selected together in order to maximise the protection offered to the destination node. If we keep the same set of parameters as previously but take λ_1 to be inversely proportional to S in Figure 5 so that the average number of protecting nodes $\lambda_1 \approx 1/S$, and the ratio of time rate to spatial rate will remain constant for any fixed value of b_1 which is the speed of motion. Figure 5 shows that there is indeed an optimum size of protection space $S = S^*$ that maximises the delay before the attacking packet can reach the destination node, and that it varies with the speed b_1 of the packet inside the protected neighbourhood. As the speed increases, the optimum size of the neighbourhood gets smaller. This may be counterintuitive but it follows from the fact that we have taken $\lambda_1 \approx 1/S$: a smaller size implies a higher "rate of protection" and hence more frequently occurring destructions of the packet which compensate for the higher speed of the packet. However the corresponding maximum values of $E[T]$ do become smaller as the packet's speed increases. In Figure 6 we set $b_1 = b_2 = 0.25$ and Λ is varied in $\lambda_1 = \Lambda/S^2$. The results are similar to the previous ones.

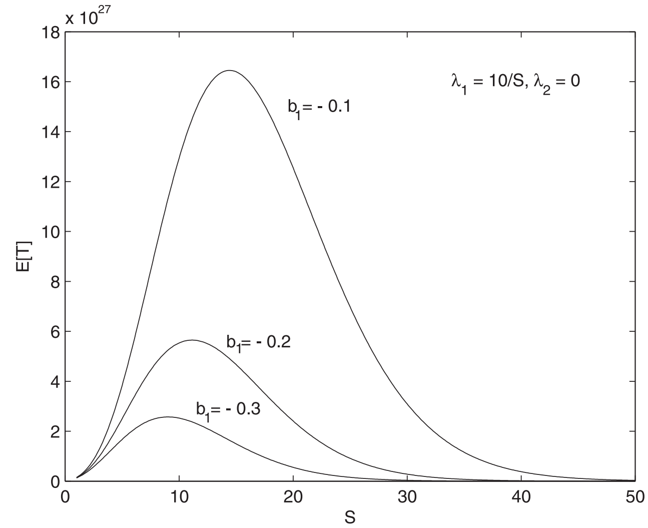

Figure 5: Average travel time $E[T]$ versus S when $\lambda_1 = 10/S$ for different values of b_1. The optimum protection area needed becomes smaller so that λ_1 increases when the packet's speed increases.

For the examples of Figures 5 and 6 the average energy expenditure is closely proportional to $E[T]$ because

$$\sum_{k=1}^{m} \int_{Z_{k-1}}^{Z_k} f_k(z)dz \cong 1$$

so that we omit showing the numerical results for the energy.

4. CONCLUSIONS

We have presented a Brownian motion model to represent a packet's travel to a destination node in a very large non-homogeneous network. A mixed analytical-numerical method has been developed to compute the average packet travel time and the energy it consumes. We observe that the degree of non-homogeneity of the network will significantly affect the average travel time and energy consumed. The role of time-outs to optimise these quantities has been exhibited, and two examples have been detailed. In the first example, packet losses (for instance due to insufficient wireless network coverage) increase as the packet reaches areas which are remote from the source and destination nodes. In the second example we model an attacking packet which may be detected and destroyed as it approaches the destination node, but in turn the attacking packet may progress more rapidly as it approaches the destination node, for instance because a directional routing being used may become more accurate. Comparing the increasing speed of approach of the packet with the possible steeper defenses of the destination node, we observe that there may be conditions whereby despite the use of time-outs the attacking packet may never make it to the destination node, while in other circumstances the attack will be successful.

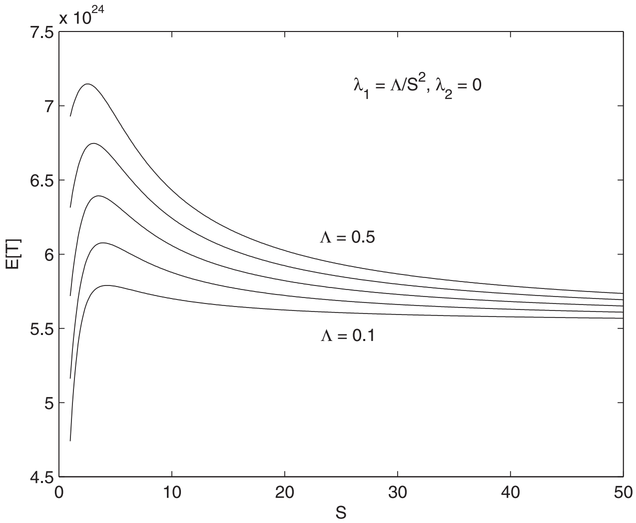

Figure 6: Average travel time $E[T]$ versus S when $\lambda_1 = \Lambda/S^2$ for $\Lambda = 0.1$ to 0.5. The protection area needed to maximise the travel time decreases as Λ increases.

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