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Wireless Sensor with Data and Energy Packets

Yasin Murat Kadioglu, *Member, IEEE* and Erol Gelenbe, *Fellow, IEEE*
Intelligent Systems and Networks Group
Dept. of Electrical and Electronic Engineering
Imperial College, London SW7 2BT, UK
{y.kadioglu14, e.gelenbe}@imperial.ac.uk

Abstract*—This paper develops a mathematical model to determine the balance of energy input and data sensing and transmission in a wireless sensing node. Since the node acquires energy through harvesting from an intermittent source, and sensing is also carried out intermittently, the node is modelled with random arrivals of both energy and data. A buffer in the node stores data packets while energy is stored in a battery acting as an energy buffer. The approach uses the "Energy Packet Network" paradigm so that both energy and data packets can be modelled as discrete quantities. We assume that for each data packet, the sensor consumes K_e energy packets for node electronics including sensing, processing, and storing and K_t energy packets for transmission. We model the node's energy and data flow by a two-dimensional random walk which represents the backlog of data and energy packets. We then simplify the model using companion matrices and matrix algebra techniques that allow us to obtain a closed-form solution for the stationary probability distribution for the random walk which allows us to compute important performance measures, including the energy consumed by the node,and its throughput in data packets transmitted as a function of the amount of power that it receives. The model also allows us to evaluate the effect of ambient noise and the needs for data retransmissions, including for the case where M sensors operate in proximity and create interference for each other.

***Index Terms*—Wireless Sensors; Energy Harvesting; Energy Packet Networks; Data Packets; Random Walk; Markov Chains; Companion Matrices.**

I. Introduction and Previous Work

A wireless sensor network (WSN) consists of nodes which sense their environment via magnetic, thermal, optical, chemical and mechanical sensors. Each node has *a sensing subsystem* for data gathering from the environment, *a processing subsystem* for local data processing and storage, and *a wireless communication system* for data receprtion and transmission [1]. WSNs can consist of a few tens to thousands of nodes, and are used in military applications, disaster management, biomedical systems, smart cities, etc. [2], [3], [4], [5], [6]. WSNs differ from traditional networks with respect to communication range, low energy consumption, limited storage and processing capability. While a few nodes can be enough for indoor environments, many nodes can be needed for outdoor environments or for industrial applications. Finite battery energy limits of wireless sensor nodes, because large batteries with long lifetime raise issues regarding size, weight and cost. Adaptive routing [7] can reduce network power consumption, but an alternative is to operate WSNs with harvested energy which can be harvested from the environment from solar and other forms of light and electromagnetic, thermal, vibrational, or piezoelectric energy conversion. [8], [9], [10]. When energy is harvested, the manner in which it is exploited and scheduled for consumption must optimize both the harvesting process and the usage [11], [12].

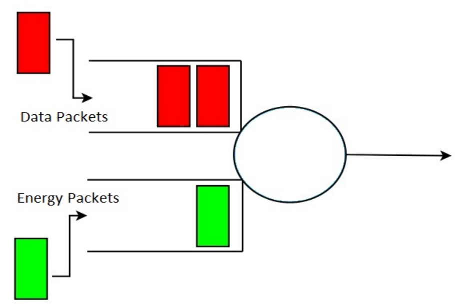

Fig. 1: Data and energy packet arrivals to the sensor node.

In [13], [14] the idea was introduced that energy storage and usage can be represented through discrete mathematical models, where an "energy packet" is an abstraction that exploits the analogy between the random arrival of harvested energy and the random arrival of packets in data networks. Recent work has modelled the joint behaviour of an energy packet buffer, i.e. a battery or capacitor, and data queue in a wireless sensor [15], [16]. Recent papers [17], [18] have exploited this paradigm to study energy and work backlog with energy harvesting and energy losses through leakage. In [19], the case where the energy required for one data packet transmission is exactly $K > 1$ energy packets was considered, assuming that energy is used only for packet transmission, not for packet processing and other sensor node electronics, while [20] has studied the model where a data packet transmission requires two energy packets: one for processing and one for packet transmission, or $K_e = K_t = 1$, in the notation of the present paper. Here we generalise the approach to arbitrary K_e and K_t values. The motivation is the that the node electronics and the transmitter may have to vary the power levels they use to deal with the speed of processing or the transmission power to overcome the effect of noise and interferenc. This generalisation then leads us to a two-dimensional random walk model which is harder to solve analytically.

II. Mathematical Model

In the mathematical model we use, data (from sensing) and energy (from energy harvesting) packets arrive from the environment at random according to two distinct and independent Poisson processes, at average rates λ and Λ, respectively. Data packets are stored in a finite capacity buffer of size B, while the energy store (battery or capacitor) has a limited capacity of E energy packets. It is assumed that a data packet can be sensed, processed and stored by the node only when there are at least K_e energy packets available in the node. Also, these K_e energy packets will be effectively expended each time a data packet is successfully received by the node. Otherwise, the data will not be received and the sensed data will go unnoticed and it will be lost. On the other hand, in order to transmit a data packet the node requires an additional number of K_t energy packets. Again all of the K_t energy packets will be consumed for one transmission. Thus the successful sensing and transmission of one data packet requires the consumption of a total of $K = K_E + K_t$ energy packets.

In our model, both the processing and transmission of a packet are assumed to occur very rapidly, if enough energy is available so that an arriving data packet is instantaneously stored if the amount of energy available is more than K_e but less than K, while it will be both stored and transmitted when the amount of energy available is at least K. Under these assumptions, we construct a two-dimensional continuous time Markov chain to represent the behaviour of the system. Let $N(t)$ and $M(t)$ be, respectively, the number of data and energy packets in the sensor node at time $t \geq 0$, so that state of the system can be represented by pair of $(N(t), M(t))$.

Let us write $p(n,m,t) = Pr[N(t) = n, M(t) = m]$ so that stationary probabilities in steady state $p(n,m) = \lim_{t\to\infty} Pr[N(t) = n, M(t) = m]$. From the above remark, we need only consider $p(n,m,t)$ for the state space S of pairs of integers $(n,m) \in S$ such that $S = \{(0,0),(n,0),(0,m),(l,k) : 1 \leq n \leq B,\ 1 \leq m \leq E,\ 1 \leq l < B, 1 \leq k < K\}$ where $K = K_e + K_t$.

In [20], the energy expended per packet for node electronics and data transmission are equal and are provided by a single energy packet, resulting in a one-dimensional Markov chain. However, when we consider the general case for K_e and K_t where they can take arbitrary values, system is no longer modeled as 1D Markov chain but 2D Markov chain in Figure 2, which makes the analysis harder. Since the energy consumption for many sensor node applications is mainly dominated by data transmission subsystem [21], we assume $K_t > K_e$. We can write the global balance equations:

$$p(0,0)[\Lambda] = \Lambda p(1,K-1) + \lambda p(0,K), \quad (1)$$

$$p(0,m)[\Lambda] = \Lambda p(0,m-1) + \lambda p(0,m+K) 1[E \geq m+K], \quad \mathbf{1 \leq m < K_e} \quad (2)$$

$$p(0,m)[\Lambda+\lambda] = \Lambda p(0,m-1) + \lambda p(0,m+K) 1[E \geq m+K], \quad \mathbf{K_e \leq m < E} \quad (3)$$

$$p(0,E)[\lambda] = \Lambda p(0,E-1), \quad (4)$$

$$p(n,0)[\Lambda] = \Lambda p(n+1,K-1) + \lambda p(n-1,K_e), \quad \mathbf{1 \leq n < B}, \quad (5)$$

$$p(n,m)[\Lambda] = \Lambda p(n,m-1) + \lambda p(n-1,m+K_e), \quad \mathbf{1 \leq n < B,\ 1 \leq m < K_e} \quad (6)$$

$$p(n,m)[\Lambda+\lambda] = \Lambda p(n,m-1) + \lambda p(n-1,m+K_e), \quad \mathbf{1 \leq n < B,\ K_e \leq m < K - K_e} \quad (7)$$

$$p(n,m)[\Lambda+\lambda] = \Lambda p(n,m-1), \quad \mathbf{1 \leq n < B,\ K - K_e \leq m < K}, \quad (8)$$

$$p(B,m)[\Lambda] = \Lambda p(B,m-1) + \lambda p(B-1,m+K_e), \quad \mathbf{K_e \leq m < K - K_e} \quad (9)$$

$$p(B,m)[\Lambda] = \Lambda\ p(B,m-1), \quad \mathbf{K - K_e \leq m < K} \quad (10)$$

$$p(B,0)[\Lambda] = \lambda\ p(B-1,K_e). \quad (11)$$

Finding a closed-form solutions for stationary probability distributions and other quantities by using these balance equations is elusive, so that we need to use different approaches for this particular system model. We can use the traditional approach where we define generator matrix Q to find the stationary probabilities. Q is an $n\times n$ matrix of n states Markov chain. In our system model, it can be easily observed that $n = E + BK + 1$. Expressing the stationary probability of each state π_i as a row vector π we can write this as a matrix equation $\pi Q = 0$.

The π_i are unknown and are the values we wish to find. Since π_i is a probability distribution we also know that the normalisation condition holds: $\sum_{x_i \in S} \pi_i = 1$. Thus, with these $n+1$ equations (global balance equations and normalisation condition) we can solve to find the n unknowns.

In order to find π_i values, we need to deal with $E+BK+2$ equations for current model, so that the complexity of the solution increases dramatically with increasing data and energy buffer sizes. To deal with the further complexity of the solution, we require spending more time and energy. Thus, apart from the traditional approach, we propose a different solution by using companion matrices and matrix algebra techniques which decreases the solution complexity.

III. Solution with Companion Matrices

For the sake of simplicity, we may merge two state indices of the sensor node by defining an one-to-one and onto function such that:

$$S_j = p(n,m) : j = nK - m + E,\ j \in \{0, 1, \cdots, BK + E\}.$$

Thus, each state (n,m) can be mapped uniquely onto states j. Next, we may consider each row of the two-dimensional Markov chain in Figure 2 as a vector V_i where $0 \leq i \leq B$, so that we will have:

$$\begin{aligned} V_0 &= [S_E, S_{E-1}, \cdots, S_1, S_0], \\ V_1 &= [S_{E+K}, S_{E+K-1}, \cdots, S_{E+2}, S_{E+1}], \\ V_2 &= [S_{E+2K}, S_{E+2K-1}, \cdots, S_{E+2+K}, S_{E+1+K}], \\ &\vdots \\ V_B &= [S_{E+BK}, \cdots, S_{E+2+BK-K}, S_{E+1+BK-K}]. \end{aligned}$$

Besides the fact that complicated state transition behaviours among the states, once we carefully observe the diagram in Figure 2, it could be realized that every row, except the first

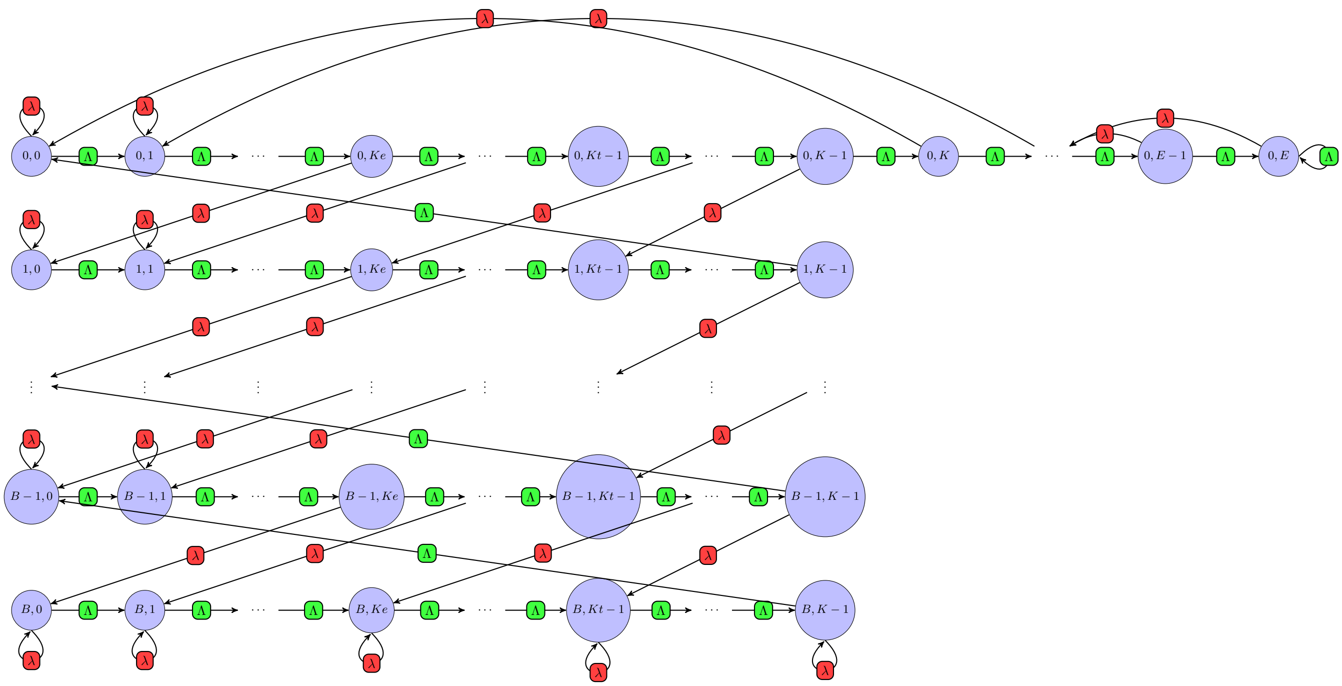

Fig. 2: Two-dimensional Markov chain state diagram representation of the system

and the last one, has exact same state transition behaviours. Therefore, we may have some recurrence relations which might reduce the number of total equations and the system complexity.

Figure 3 shows the state representation of i^{th} row of the two-dimensional state diagram or vector V_i where $0 < i < B$. We observe in Figure 3 that for vector V_i, there are 3 different transition behaviours among the states so that we can subdivide the vector into 3 separate regions by which we can write following equations:

- *For Region1,* $0 \leq m < K_e$*:*

$$S_{N+1} = S_N - (\frac{\lambda}{\Lambda})\ S_{N-K-K_e}, \tag{12}$$

- *For Region2,* $K_e \leq m < K_t$*:*

$$S_{N+1} = S_N + (\frac{\lambda}{\Lambda})\ (S_N - S_{N-K-K_e}), \tag{13}$$

- *For Region3,* $K_t \leq m < K$ *:*

$$S_{N+1} = (1 + \frac{\lambda}{\Lambda})\ S_N. \tag{14}$$

Note that the (12) and (13) are linearly recursive sequence of order $K + K_e + 1$ whose minimum value is 8. We know that there is no solution in radicals to the general polynomial equations of degree 5 and more by Abel&Ruffini theorem [22]. Thus, it is not easy to solve these equations and express a closed-form solution for stationary probability distributions. However, we may use companion matrices of each equation to express transitions among states. To provide consistency among the companion matrices, we will consider each one as a square matrix with dimension $K + K_e + 1$. Once we consider the vector V_1, we can write state transitions by using companion matrices as follows:

$$\begin{bmatrix} S_{E+2} \\ S_{E+1} \\ \vdots \\ S_{E+2-K-K_e} \end{bmatrix} = \begin{bmatrix} (1+\frac{\lambda}{\Lambda}) & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} S_{E+1} \\ S_E \\ \vdots \\ S_{E+1-K-K_e} \end{bmatrix}$$

or equivalently $\overrightarrow{S_{E+2}} = C_3\ \overrightarrow{S_{E+1}}$.

Other state vectors in *Region3* can also be expressed iteratively as follows:

$$\begin{aligned} \overrightarrow{S_{E+3}} &= C_3\ \overrightarrow{S_{E+2}} = C_3^2\ \overrightarrow{S_{E+1}}, \\ \overrightarrow{S_{E+4}} &= C_3\ \overrightarrow{S_{E+3}} = C_3^3\ \overrightarrow{S_{E+1}}, \\ &\vdots \\ \overrightarrow{S_{E+K_e+1}} &= C_3\ \overrightarrow{S_{E+K_e}} = C_3^{K_e}\ \overrightarrow{S_{E+1}}. \end{aligned}$$

Similarly, for *Region2*:

$$\begin{aligned} \overrightarrow{S_{E+K_e+2}} &= C_2\ \overrightarrow{S_{E+K_e+1}} = C_2\ C_3^{K_e}\ \overrightarrow{S_{E+1}}, \\ \overrightarrow{S_{E+K_e+3}} &= C_2\ \overrightarrow{S_{E+K_e+2}} = C_2^2\ C_3^{K_e}\ \overrightarrow{S_{E+1}}, \\ &\vdots \\ \overrightarrow{S_{E+K_t+1}} &= C_2\ \overrightarrow{S_{E+K_t-1}} = C_2^{K_t-K_e}\ C_3^{K_e}\ \overrightarrow{S_{E+1}}, \end{aligned}$$

and for *Region1*:

$$\begin{aligned} \overrightarrow{S_{E+K_t+2}} &= C_1\ \overrightarrow{S_{E+K_t+1}} = C_1\ C_2^{K_t-K_e}\ C_3^{K_e}\ \overrightarrow{S_{E+1}}, \\ \overrightarrow{S_{E+K_t+3}} &= C_1\ \overrightarrow{S_{E+K_t+2}} = C_1^2\ C_2^{K_t-K_e}\ C_3^{K_e}\ \overrightarrow{S_{E+1}}, \\ &\vdots \\ \overrightarrow{S_{E+K}} &= C_1\ \overrightarrow{S_{E+K-1}} = C_1^{K_e-1}\ C_2^{K_t-K_e}\ C_3^{K_e}\ \overrightarrow{S_{E+1}}, \\ \overrightarrow{S_{E+K+1}} &= C_1\ \overrightarrow{S_{E+K}} = C_1^{K_e}\ C_2^{K_t-K_e}\ C_3^{K_e}\ \overrightarrow{S_{E+1}}, \end{aligned}$$

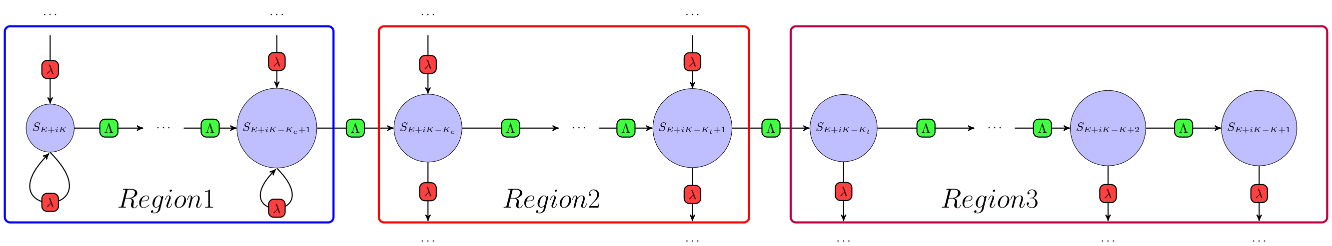

Fig. 3: State diagram representation of the vector V_i

where

$$C_1 = \begin{bmatrix} 1 & 0 & \dots & 0 & -\frac{\lambda}{\Lambda} \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} (1+\frac{\lambda}{\Lambda}) & 0 & \dots & 0 & -\frac{\lambda}{\Lambda} \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

We showed that we can express all the state vectors in V_1 in terms of $\overrightarrow{S_{E+1}}$ and companion matrices. In fact, the same procedure can be followed for the other row vectors and any state vector $\overrightarrow{S_N} : S_N \in V_i, 0 < i < B$ can be expressed as:

$$\overrightarrow{S_N} = \begin{cases} {C_3}^{\alpha}\ C^{\lfloor \frac{N-E-1}{K} \rfloor}\ \overrightarrow{S_{E+1}} & 0 \le \alpha \le K_e \\ {C_2}^{\alpha-K_e}\ {C_3}^{K_e}\ C^{\lfloor \frac{N-E-1}{K} \rfloor}\ \overrightarrow{S_{E+1}} & K_e < \alpha \le K_t \\ {C_1}^{\alpha-K_t}\ {C_2}^{K_t-K_e}\ {C_3}^{K_e}\ C^{\lfloor \frac{N-E-1}{K} \rfloor}\ \overrightarrow{S_{E+1}} & K_t < \alpha < K \end{cases} \tag{15}$$

where $\lfloor \cdot \rfloor$ is a function that returns the largest integer less than or equal to its argument, C is the multiplication of companion matrices, i.e., $C = C_1^{K_e}\ C_2^{K_t-K_e}\ C_3^{K_e}$, and the parameter $\alpha = (N-E+K-1) \pmod K$. Thus, we are able to express the state vectors $\overrightarrow{S_N} : S_N \in V_i, 0 < i < B$ with respect to companion matrices and the state vector $\overrightarrow{S_{E+1}}$.

After studying on the majority of the vectors, we can now consider V_0 and write following characteristic equations:

- *For* $0 < N \le E - K + 1$*:*

$$S_N = (\frac{\lambda}{\Lambda})(1+\frac{\lambda}{\Lambda})^{N-1} S_0 \tag{16}$$

- *For* $E - K + 1 < N \le E - K_e$*:*

$$S_{N+1} = (1+\frac{\lambda}{\Lambda})S_N - \frac{\lambda}{\Lambda} S_{N-K} \tag{17}$$

- *For* $E - K_e < N \le E$*:*

$$S_{N+1} = S_N - \frac{\lambda}{\Lambda} S_{N-K} \tag{18}$$

Thus, we can express $\overrightarrow{S_{K+1}}$ by companion matrix from equation (17):

$$\begin{bmatrix} S_{K+1} \\ S_K \\ \vdots \\ S_{1-K_e} \end{bmatrix} = \begin{bmatrix} (1+\frac{\lambda}{\Lambda}) & 0 & \dots & 0 & -\frac{\lambda}{\Lambda} \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} S_K \\ S_{K-1} \\ \vdots \\ S_{-K_e} \end{bmatrix}$$

where the states $\{S_j : j < 0\}$ are redundant, i.e., probability of these states is zero, and they are used to provide consistency among dimensions of companion matrices.

Thus, we may keep the iteration and express other state vectors as follows:

$$\begin{aligned} \overrightarrow{S_{K+1}} &= C_2\ \overrightarrow{S_K}, \\ \overrightarrow{S_{K+2}} &= C_2\ \overrightarrow{S_{K+1}} = C_2^2\ \overrightarrow{S_K}, \\ &\vdots \\ \overrightarrow{S_{E-K_e+1}} &= C_2\ \overrightarrow{S_{E-K_e}} = C_2^{E+1-(K+K_e)}\ \overrightarrow{S_K}. \end{aligned}$$

and

$$\begin{aligned} \overrightarrow{S_{E-K_e+2}} &= C_1\ \overrightarrow{S_{E-K_e+1}} = C_1\ C_2^{E+1-(K+K_e)}\ \overrightarrow{S_K}, \\ \overrightarrow{S_{E-K_e+3}} &= C_1\ \overrightarrow{S_{E-K_e+2}} = C_1^2\ C_2^{E+1-(K+K_e)}\ \overrightarrow{S_K}, \\ &\vdots \\ \overrightarrow{S_{E+1}} &= C_1\ \overrightarrow{S_E} = C_1^{K_e}\ C_2^{E+1-(K+K_e)}\ \overrightarrow{S_K}. \end{aligned}$$

We may also express $\overrightarrow{S_K}$ by using (16) as:

$$\overrightarrow{S_K} = \begin{bmatrix} S_K \\ S_{K-1} \\ \vdots \\ S_2 \\ S_1 \\ S_0 \\ S_{-1} \\ \vdots \\ S_{-K_e} \end{bmatrix} = \frac{\lambda}{\Lambda} S_0 \begin{bmatrix} (1+\frac{\lambda}{\Lambda})^{K-1} \\ (1+\frac{\lambda}{\Lambda})^{K-2} \\ \vdots \\ (1+\frac{\lambda}{\Lambda}) \\ 1 \\ \frac{\Lambda}{\lambda} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = (\frac{\lambda}{\Lambda}) S_0 \overrightarrow{\gamma}$$

Thus, we may write:

$$\overrightarrow{S_N} = \begin{cases} {C_2}^{N-K}\ (\frac{\lambda}{\Lambda}) S_0 \overrightarrow{\gamma} & K \le N \le E - K_e + 1 \\ {C_1}^{N-\varsigma_1}\ {C_2}^{\varsigma_2}\ (\frac{\lambda}{\Lambda}) S_0 \overrightarrow{\gamma} & E - K_e + 1 < N \le E + 1 \end{cases} \tag{19}$$

where $\varsigma_1 = E - K_e + 1$ and $\varsigma_2 = \varsigma_1 - K$.

We also may replace $\overrightarrow{S_{E+1}}$ in (15) and rewrite it as:

$$\overrightarrow{S_N} = \begin{cases} {C_3}^{\alpha}\ C^{\lfloor \frac{N-E-1}{K} \rfloor} C' & 0 \le \alpha \le K_e \\ {C_2}^{\alpha-K_e}\ {C_3}^{K_e}\ C^{\lfloor \frac{N-E-1}{K} \rfloor} C' & K_e < \alpha \le K_t \\ {C_1}^{\alpha-K_t}\ {C_2}^{K_t-K_e}\ {C_3}^{K_e}\ C^{\lfloor \frac{N-E-1}{K} \rfloor} C' & K_t < \alpha < K \end{cases} \tag{20}$$

where $C' = C_1^{K_e} C_2^{E+1-(K+K_e)} (\frac{\lambda}{\Lambda}) S_0 \overrightarrow{\gamma}$.

Also, we may write following characteristic equations for the states in V_B:

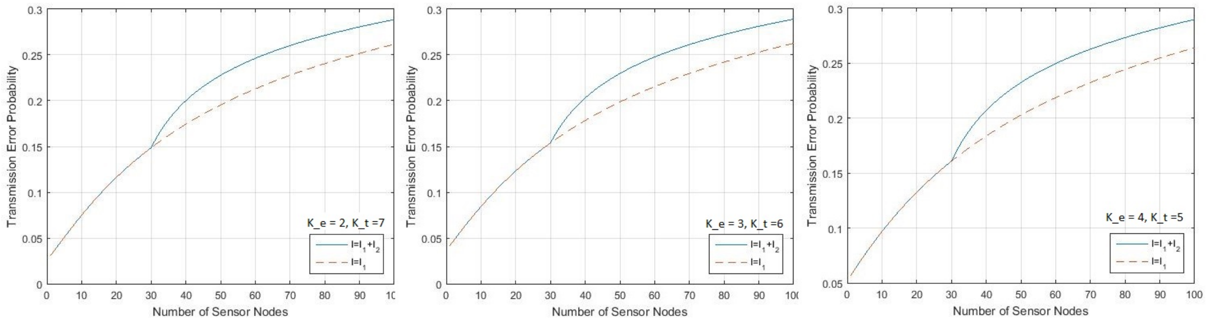

Fig. 4: Receiving error probabilities with same K and different K_e & K_t values

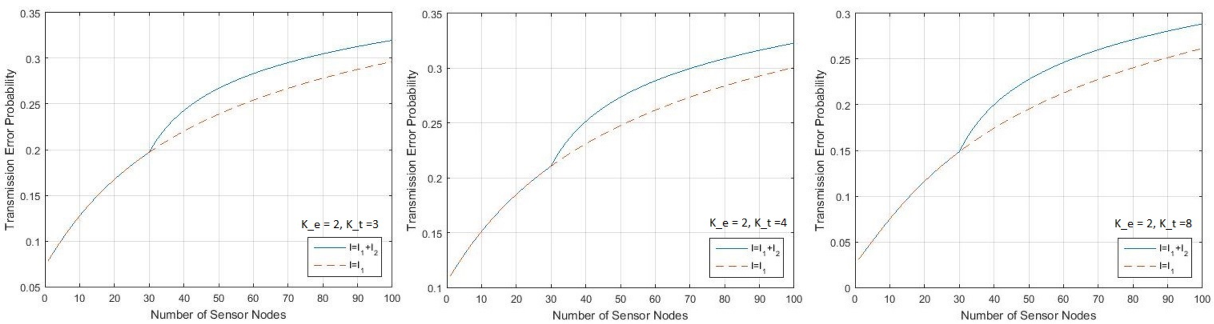

Fig. 5: Receiving error probabilities with increasing K_t and K values

- *For* $BK+E-K+1 \leq N \leq BK+E-K_t$*:*

$$S_{N+1} = S_N \tag{21}$$

- *For* $BK+E-K_t < N < BK+E$*:*

$$S_{N+1} = S_N - \frac{\lambda}{\Lambda} S_{N-K-K_e} \tag{22}$$

Therefore, we may write followings:

$$\overrightarrow{S_{BK+E-K_t+1}} = \cdots = \overrightarrow{S_{BK+E+1-K}}$$

and

$$\begin{aligned} \overrightarrow{S_{BK+E-K_t+2}} &= C_1 \ \overrightarrow{S_{BK+E+1-K}}, \\ \overrightarrow{S_{BK+E-K_t+3}} &= C_1^2 \ \overrightarrow{S_{BK+E+1-K}}, \\ &\vdots \\ \overrightarrow{S_{BK+E}} &= C_1^{K_t-1} \ \overrightarrow{S_{BK+E+1-K}}. \end{aligned}$$

We can express the state vector $\overrightarrow{S_{BK+E+1-K}}$ from (20) as,

$$\overrightarrow{S_{BK+E+1-K}} = C^{B-1}C'$$

Therefore, we may write $\overrightarrow{S_N} : S_N \in V_B$:

$$\overrightarrow{S_N} = \begin{cases} C^{B-1}C' & BK+E+1-K \leq N \leq BK+E+1-K_t \\ {C_1}^{N-(BK+E-K_t+1)} C^{B-1}C' & BK+E-K_t+1 < N \leq BK+E \end{cases} \tag{23}$$

Thus, we can express any state vector $\overrightarrow{S_N} : \{S_N : 0 \leq N \leq BK+E\}$ as a function of companion matrices C_1, C_2, C_3, required energy packet amounts K_t and K_e, the vector $\overrightarrow{\gamma}$, and the state S_0 by combining equations (19), (20), (23). We also know that the normalisation condition holds $\sum_{i=0}^{BK+E} S_i = 1$, so that we can find stationary probability distribution of S_0 and of all the other states in the system.

IV. Numerical Results

Since the rate of energy is in power units, the average total power consumed by the sensor node is $\xi = (1-S_0)\Lambda$, where the reduction $S_0\Lambda$ is due to the lost energy packets when the battery or capacitor is full and it overflows. On the other hand, the average radiated power is $\phi = \kappa\xi$ where $\kappa = \frac{K_t}{K}$. If there are M identical sensors operating at the same power level [20] using BPSK transmission, the probability that a bit is received correctly is given by:

$$Q\left(\sqrt{\frac{\eta K_t}{\eta\kappa\xi\zeta\ (M-1) + \eta\kappa\xi\ (\frac{M-M'}{M})1[M > M'] + N}}\right), \tag{24}$$

where the denominator stands for interference $I_1 + I_2$ such that $I_1 = \eta\kappa\xi\zeta\ (M-1)$ and $I_2 = \eta\kappa\xi\ (\frac{M-M'}{M})1[M > M']$, plus the noise power level denoted by N. M' is the number of separate frequency channels that transmission can occur so that whenever the number of sensor nodes exceeds M', there will be an additional interference level I_2. Also, η is the reduction of transmitted power that is received at the receiver, ζ is a factor representing the effect of side-band frequency channels among the M' separate frequency channels and it is typically much smaller than 1, and $Q(x) = \frac{1}{2}[1 - erf(\frac{x}{\sqrt{2}})]$.

In a simple system with the hypothetical parameters $\eta = 0.5$, $\zeta = 0.02$, $M' = 30, N = 1$, and $E = 10, B = 3, \Lambda = 10, \lambda = 2$ for Figure 4 and Figure 5. In Figure 4, we observe the effect of the number of sensor nodes on the receiver error probability with same K value and different K_e and K_t values where we assume $K = 9$ and $K_e = 2, 3, 4$ & $K_t = 7, 6, 5$, respectively. Although there is a slight increase on transmission errors with decreasing K_t values, the overall error characteristic is almost same with the same K value despite the different K_e and K_t values. On the other hand, In Figure 5, we observe the effect of the number of sensor nodes on the receiver error probability with increasing K values where we assume $K_e = 2$ and $K_t = 3, 4, 8$, respectively. The increasing K values results in smaller errors.

V. Conclusions

This paper analyses wireless sensor nodes that harvest energy and sense data from the environment. Each node stores and transmits data in the form of discrete packets. Our approach uses a discrete representation of energy and data packets leading to a Markov chain representation which is computationally cumbersome and does not readily leads to an analytical solution. Thus we propose a solution that uses companion matrices and linear algebra to reduce the model's computational complexity. We also exploit certain regularity properties of the matrix structure resulting in efficient numerical computation of all the metrics of interest. In particular we obtain the steady-state distribution of the backlog of data and energy packets, the system throughput in terms of successfully transmitted packets, and the possible loss of energy when the energy storage device is full and energy is harvested. The analysis also allows us to include the effect of the communication environment where noise, and the interference due to multiple statisticallly identical wireless sensor nodes, will result in increased data errors and reduced effective throughput.

Future work will address a more practical system where non-transmitted data may be lost due to time-outs, and energy may be lost through leakage. We also plan to consider related multi-hop and networked systems.

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References

[1] G. Anastasi, M. Conti, M. D. Francesco, and A. Passarella, "Energy conservation in wireless sensor networks: A survey," *Ad Hoc Networks*, vol. 7, no. 3, pp. 537 – 568, 2009.

[2] J. Yick, B. Mukherjee, and D. Ghosal, "Wireless sensor network survey," *Computer Networks*, vol. 52, no. 12, pp. 2292 – 2330, 2008.

[3] J. Yick, B. Mukherjee, and D. Ghosal, "Analysis of a prediction-based mobility adaptive tracking algorithm," in *Broadband Networks, 2005. BroadNets 2005. 2nd International Conference on*, pp. 753–760 Vol. 1, Oct 2005.

[4] M. Castillo-Effer, D. Quintela, W. Moreno, R. Jordan, and W. Westhoff, "Wireless sensor networks for flash-flood alerting," in *Devices, Circuits and Systems, 2004. Proceedings of the Fifth IEEE International Caracas Conference on*, vol. 1, pp. 142–146, Nov 2004.

[5] T. Gao, D. Greenspan, M. Welsh, R. Juang, and A. Alm, "Vital signs monitoring and patient tracking over a wireless network," in *Engineering in Medicine and Biology Society, 2005. IEEE-EMBS 2005. 27th Annual International Conference of the*, pp. 102–105, Jan 2005.

[6] G. Werner-Allen, K. Lorincz, M. Welsh, O. Marcillo, J. Johnson, M. Ruiz, and J. Lees, "Deploying a wireless sensor network on an active volcano," *IEEE Internet Computing*, vol. 10, pp. 18–25, Mar. 2006.

[7] E. Gelenbe and C. Morfopoulou, "A framework for energy-aware routing in packet networks," *The Computer Journal*, vol. 54, no. 6, pp. 850–859, 2011.

[8] W. K. Seah, Z. A. Eu, and H.-P. Tan, "Wireless sensor networks powered by ambient energy harvesting (wsn-heap)-survey and challenges," in *Wireless Communication, Vehicular Technology, Information Theory and Aerospace & Electronic Systems Technology, 2009. Wireless VITAE 2009. 1st International Conference on*, pp. 1–5, IEEE, 2009.

[9] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 1180–1189, 2012.

[10] J. Jeon and A. Ephremides, "On the stability of random multiple access with stochastic energy harvesting," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 3, pp. 571–584, 2015.

[11] J. Yang and S. Ulukuş, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. Commun.*, vol. 60, pp. 220–230, 2012.

[12] E. Gelenbe, D. Gesbert, D. Gündüz, H. Külah, and E. Uysal-Biyikoglu, "Energy harvesting communication networks, optimization and demonstration: The e-crops project," in *Proc. 24th Tyrrhenian International Workshop 2013 on Digital Communications—Green ICT (TIWDC)*, (Genoa, Italy), pp. 1–6, IEEE, 23–25 September 2013.

[13] E. Gelenbe, "Energy packet networks: Ict based energy allocation and storage - (invited paper)," in *GreeNets* (J. J. P. C. Rodrigues, L. Zhou, M. Chen, and A. Kailas, eds.), vol. 51 of *Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering*, pp. 186–195, Springer, 2011.

[14] E. Gelenbe, "Energy packet networks: adaptive energy management for the cloud," in *Proceeding CloudCP '12, Proceedings of the 2nd International Workshop on Cloud Computing Platforms*, ACM, 2012.

[15] E. Gelenbe, "A sensor node with energy harvesting," *ACM SIGMETRICS Performance Evaluation Review*, vol. 42, no. 2, pp. 37–39, 2014.

[16] E. Gelenbe, "Synchronising energy harvesting and data packets in a wireless sensor," *Energies*, vol. 8, no. 1, pp. 356–369, 2015.

[17] E. Gelenbe and Y. M. Kadioglu, "Performance of an autonomous energy harvesting wireless sensor," in *Information Sciences and Systems*, pp. 35–43, Springer, 2016.

[18] E. Gelenbe and Y. M. Kadioglu, "Energy loss through standby and leakage in energy harvesting wireless sensors," in *20th IEEE International Workshop on Computer Aided Modelling and Design of Communication Links and Networks,*, 2015.

[19] Y. M. Kadioglu and E. Gelenbe, "Packet transmission with k energy packets in an energy harvesting sensor," in *Proceedings of the 2Nd International Workshop on Energy-Aware Simulation*, ENERGY-SIM '16, (New York, NY, USA), pp. 1:1–1:6, ACM, 2016.

[20] Y. M. Kadioglu, "Energy consumption model for data processing and transmission in energy harvesting wireless sensors," in *Computer and Information Sciences - 31st International Symposium, ISCIS 2016, Kraków, Poland, October 27-28, Proceedings*, pp. 117–125, 2016.

[21] M. Calle and J. Kabara, "Measuring energy consumption in wireless sensor networks using gsp," in *Personal, Indoor and Mobile Radio Communications, 2006 IEEE 17th International Symposium on*, pp. 1–5, Sept 2006.

[22] H. Żołądek, "The topological proof of abel–ruffini theorem," *Topol. Methods Nonlinear Anal*, vol. 16, pp. 253–265, 2000.