

A Diffusion Model for Packet Travel Time in a Random Multi-Hop Medium

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We consider a wireless network in which packets are forwarded opportunistically from the source towards the destination, without accurate knowledge of the direction that they should take. A Brownian Motion model that includes the effect of packet losses, and subsequent retransmission after a time-out, is used to compute the average travel time of the packet. We show that the average packet travel time can be minimised by a judicious choice of the time-out, and its optimum value in turn depends on other system parameters such as packet loss probabilities. We present simulations that illustrate the analytical results.

Categories and Subject Descriptors: C.2 [**Computer Communication Networks**]: Packet Routing; C.4 [**Performance of Systems**]: Analytical Models; G.3 [**Stochastic Processes**]: Diffusion Approximations

General Terms: Wireless Networks, Ad Hoc Networks

Additional Key Words and Phrases: Sensor Networks, Autonomic Communications, Packet Travel Time, Simulation

1. INTRODUCTION

In wireless ad hoc networks or sensor networks [Perkins 2000; Akyildiz et al. 2002], uncertainties in the physical location of nodes, their mobility and the characteristics of the wireless medium, as well as possible physical obstacles and the algorithms that are used to convey packets from source to destination, all have a significant impact on the the time it takes for packets to arrive. Because of the uncertainty in these transmission media, much attention has been devoted to developing reliable techniques for packet forwarding in these complex environments [Intanagonwiwawat et al. 2000; Krishnamachari et al. 2002].

We consider a packet wireless network in which nodes are distributed over an area or volume of space, but where we do not know about the presence, the exact location, or the reliability of nodes. In such a system, packets may typically travel over a random number of multiple hops before they reach the destination. A node that hears a packet transmission from some neighbour, may inform the neighbour

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that it has received it. In turn the transmitting node will inform its neighbours that it has selected a particular recipient node. Although the transmitting node may try to select the best among the recipients based on its own estimate of the neighbour which is geographically closest to the final destination, a transmission step may actually move the packet further away from the destination if at some step a better option is unavailable. At some point on its path, it may also happen that a packet cannot be forwarded any further, or that the intermediate node has a failure, or that the packet is lost through noise or some other transient effect. In that case, the packet may be retransmitted after some time-out period has elapsed, either by the source or from some intermediate storage location on the path that it traversed before it was lost.

More specifically, consider a source A that wishes to send a packet to a fixed receiver B which is placed at some physical distance J from the source. Each of the nodes in the network have a wireless range of d and we assume that $J \gg d$ so that multiple hops need to be used to reach B . From now on we replace the physical distance d by a distance in terms of the least possible number of hops $D = J/d$, and we are assuming the $D \gg 1$. Furthermore we assume that some intermediate nodes may be out of range of each other, or that nodes may have failed. In this general setting, the problem we address is that of estimating the time T_1 it takes a packet to travel from A to B given a known value of the initial distance D . This question has been considered by several authors [Zorzi and Rao 2003; Shakkottai et al. 2003; Fotoh et al. 2005], and is relevant both to the evaluation of ad hoc network performance [Ko and Vaidya 1998; Royer and Toh 1999; Perkins 2000] and to wireless sensor networks [Intanagonwiwawat et al. 2000; Akyildiz et al. 2002; Krishnamachari et al. 2002]. It is also a mathematically interesting and very difficult problem, even when simplifying assumptions can be made about the location of nodes [Shakkottai et al. 2003; Fotoh et al. 2005] which so far has not been solved in an exact manner, so that the results that are available only offer bounds and approximations for the travel time or the number of hops that the packet traverses. It can also be of practical interest to wired networks which use local search techniques for packet routing [Gelenbe et al. 2001; Gelenbe et al. 2004; Gelenbe et al. 2004] rather than deterministic shortest-path techniques.

In this paper we use a mathematical approach based on diffusion approximations to estimate the amount of time it will take a packet to go from its source to the destination in the presence of such random effects along the path. The practical context we have in mind is that the packet is being forwarded using geographical routing [Ko and Vaidya 1998] with randomising effects due to the location of the relay nodes, possible physical obstacles, and node failures, and with the possibility that each relay node is smart enough to select the “best” next node among all those that may have responded positively when the packet is transmitted. We do not consider the possibility of the same packet being forwarded by many nodes at the same time; in other words we exclude the possibility of many-to-one or one-to-many forwarding techniques. Such approaches can improve the robustness of the communication, but they can lead to excessive power consumption and may also lead to excessive congestion in the network. In Section 5 we complete the paper by presenting several simulation results that illustrate the theoretical analysis.

2. MODELLING THE PACKET TRAVEL TIME

The physical medium we are modelling may be quite complex. For instance, if this is a geographic terrain in which packets are sent by radio wireless, the nodes used to forward the packet will in general not be placed at precise regular intervals. The signal propagation may be perturbed or obstructed by physical objects which may deflect the signal, absorb or reflect it. Furthermore, when a node that needs to forward a packet does find a neighbour within transmission range, it may choose the neighbour to which it forwards the packet based on a variety of criteria including the quality of the communication link or on the basis of its estimate of which is the “best” neighbour it can reach. Alternatively it may choose to broadcast the packet to all neighbours within range, letting the neighbours themselves decide who wants to accept and forward the packet further. All of these factors introduce randomness in the amount of time it will take a packet to travel one hop and then to reach its destination.

Thus rather than making assumptions about the nature of the medium, its geometry, and the location of the nodes that are used as relays on the path from the source to the destination, we consider the instantaneous distance Y_t of the packet at time $t \geq 0$ to its destination, with $Y_0 = D$. The total travel time T_1 :

$$T_1 = \inf\{t : Y_t = 0\}. \quad (1)$$

We denote the process representing the distance of the packet to its destination by $Y = \{Y_t : 0 \leq t < T_1\}$. At time T_1 the packet’s travel will stop if $Y_t = 0$ for $t \geq T_1$. We will assume that when $Y_t = y > 0$ but $t < T_1$, the following events can occur in the time interval $[t, t + \Delta t]$:

- With probability $\lambda\Delta t + o(\Delta t)$ the packet is lost, either because there are no nodes to which the current node can forward the packet, or because of some transmission failure or error. It may also correspond to an absorption or strong attenuation of the signal due to obstacles, or to the fact that the closest relay node is out of range.
- With probability $1 - \lambda\Delta t + o(\Delta t)$ the packet is received by a relay node whose distance from the destination is $Y_{t+\Delta t}$.

Let $b\Delta t$ represent the mean change over time $[t, t + \Delta t]$ of the distance of the packet to the destination, and let the variance of the distance travelled by the packet in the same time interval be $c\Delta t$, where b and c are constants:

$$b = \lim_{\Delta t \rightarrow 0} \frac{E[Y_{t+\Delta t} - Y_t | Y_t = y]}{\Delta t},$$

$$c = \lim_{\Delta t \rightarrow 0} \frac{E[(Y_{t+\Delta t} - Y_t)^2] - (E[Y_{t+\Delta t} - Y_t | Y_t = y])^2}{\Delta t}.$$

If b and c do not depend on y and t , we are in fact assuming that the propagation medium and the distribution of the relay nodes are homogenous in space and that the system characteristics do not change over time.

In the next section we first discuss how we transform the transient process of “going from the source to the destination just once” to an ergodic process where “the packet goes from the source to the destination, stops there for a short time, and

then the travel restarts and is repeated indefinitely". This transformation allows us to use the steady-state distributions of the ergodic process so as to compute the average travel time it takes for the packet to go from the source to the destination. Thus we will replace the computation of the first passage time T_1 in the process Y by a simpler analysis which will yield the expected value $E[T_1]$ from an ergodic process Z which is defined below.

3. CONSTRUCTION OF AN ERGODIC PROCESS

For the sake of simplicity, let us first suppose that the packet does not get lost on its path, so that $\lambda = 0$. The more general case will be handled later.

Consider a new random process $Z = \{Z_t : t \geq 0\}$ which is identical in sample path to Y until time T_1 . After T_1 the process Z will reside at the point $z = 0$ for a random time H_1 , after which Z jumps to point D and then stochastically repeats its previous behaviour indefinitely. Suppose that at some time $t = T_i$ the destination is reached, i.e. $Z(T_i) = 0$; then after a random time H_i we assume that the search process starts again, so that at $t = T_i + H_i$ the process Z jumps back to the starting point of the search and $Z(T_i + H_i^+) = D$, and the search process is re-initialised. This process repeats itself indefinitely.

Let H_i , $i = 1, 2, \dots$, be independent and identically distributed positive random variables, and $T_{i+1} = \inf\{t : T_{i+1} > T_i, Z_{T_{i+1}} = 0\}$ for $i = 1, 2, \dots$. Then Z will have the renewal property:

$$P[Z(t) > z] = P[Z(t + T_i + H_i) > z] \quad (2)$$

for any $t \geq 0$, $z \geq 0$; the instants $\{T_i + H_i\}$ are renewal instants of the process Z for $i \geq 1$.

Remark 1 Let $E[T] = E[T_i]$ for any i , and assume that $E[H_i] = 1$. Let

$$P = \lim_{t \rightarrow \infty} P[Z(t) = 0]. \quad (3)$$

Then:

$$E[T] = \frac{1}{P} - 1. \quad (4)$$

Proof This follows from the fact that the process $\{X(t), t \geq 0\}$ defined by $X(t) = 1[Z(t)]$ is a two state (0 and 1) semi-Markov process¹. It is easy to see that $P[X(t) = 0] = P[Z(t) = 0]$ so that $P = \lim_{t \rightarrow \infty} P[Z(t) = 0] = \lim_{t \rightarrow \infty} P[X(t) = 0]$. Since

$$\lim_{t \rightarrow \infty} P[X(t) = 0] = \frac{E[H]}{E[H] + E[T]}, \quad (5)$$

where $E[H] = E[H_i]$ for any i . Since $E[H] = 1$, the result follows. **QED**

We represent the distance of the packet to the destination by a Brownian motion [Einstein 1926; Medhi 1991]. In order to take into account the artificial holding time at the boundary $z = 0$ described above, the process Z will be represented as a

¹We use $1[y]$ to denote the usual characteristic function $1[y] = 1$ if $y > 0$, and $1[y] = 0$ otherwise.

Brownian motion modified to have holding times at the boundary $z = 0$, from which it jumps to an interior point (the starting point of travel at $z = D$) as suggested in some earlier work on diffusion approximations to queues [Gelenbe 1975].

The holding time at the boundary $z = 0$ corresponds to the artificial time H_i spent by the packet at $z = 0$ before it once again starts the search process. The new start of the search is represented by an instantaneous jump of the process Z from $z = 0$ to $z = D$. Thus in addition to the usual diffusion equation, the process we consider will have a discrete (i.e. not continuous) component as described below.

In summary, the process Z representing the distance of the search packet, or of the searcher, from its destination at time t is characterised by the average change in distance b to the destination per unit time, and by the variance in the change in distance per unit time c .

3.1 The diffusion equation

We will now express the process Z completely so as to obtain P . We first write the equations that the probability density function $f_{z,t}dz = P[z \leq Z(t) < z + dz]$, $z > 0$, and the probability mass $P(t)$, $t \geq 0$ must satisfy. Notice that:

$$\frac{\partial f_{z,t}}{\partial t} \approx \frac{P[z \leq Z(t + \Delta t) < z + dz] - P[z \leq Z(t) < z + dz]}{\Delta t}, \quad (6)$$

and $P(t) = P[Z(t) = 0]$ is the probability that the packet is at the destination point at time t . At $z = 0$ we set $f_{0,t} = 0$ so that only the mass $P(t)$ has an effect at that point.

Clearly the packet is either travelling in search of the destination, or it is at rest at $z = 0$ waiting to restart a travel time. Hence for any t :

$$1 = P(t) + \int_0^\infty f_{z,t} dz. \quad (7)$$

In the process Z the packet stays at the destination for an average of one time unit (i.e it leaves that point at rate 1) and when it leaves it, it will jump to the point $z = D$ restarting the search process towards the destination. Thus we have a Dirac function term $P(t)\delta(z - D)$ being added to the time derivative of the density function in the diffusion equation, since the packet leaves the rest state at unit rate:

$$\frac{\partial f_{z,t}}{\partial t} = -b \frac{\partial f_{z,t}}{\partial z} + \frac{1}{2}c \frac{\partial^2 f_{z,t}}{\partial z^2} + P(t)\delta(z - D) \quad (8)$$

The first two terms on the right hand side of the expression (8) are the classical terms that are obtained when one uses a continuous diffusion approximation for a discrete process (for a detailed derivation of the diffusion approximation of a discrete state-space process see pp. 221-226 of [Medhi 1991]).

Taking the (partial) time derivative of (7) we have:

$$\frac{d}{dt} P(t) = - \int_{0^+}^\infty \frac{\partial f_{z,t}}{\partial t} dz, \quad (9)$$

and using (8) we get

$$\int_{0^+}^\infty \frac{\partial f_{z,t}}{\partial t} dz = [-b \frac{\partial f_{z,t}}{\partial z} + \frac{1}{2}c \frac{\partial f_{z,t}}{\partial z}]_{0^+}^\infty + P(t) \quad (10)$$

so that

$$\frac{d}{dt}P(t) = -P(t) - [-bf_{z,t} + \frac{1}{2}c\frac{\partial f_{z,t}}{\partial z}]|_{0^+}^{\infty}. \quad (11)$$

We know that $\lim_{z \rightarrow 0^+} f_{z,t} = 0$, and furthermore because $f_{z,t}$ is a probability density function, we must have $\lim_{z \rightarrow \infty} f_{z,t} = 0$ and $\lim_{z \rightarrow \infty} \frac{\partial f_{z,t}}{\partial z} = 0$. Therefore we remain with:

$$\frac{d}{dt}P(t) = -P(t) + \frac{1}{2}c\frac{\partial f_{z,t}}{\partial z}|_{0^+}^{\infty}, \quad (12)$$

In order to obtain P and hence $E[T]$, we seek the stationary solution of equations (8), (12) with the constraint (7), by setting the time derivatives to zero. To obtain the stationary solution of the density function f_z for $z > D$, we integrate (8). We note that if the sum of the probabilities as indicated in (7) is to remain finite with $b < 0$, then the only possible form is:

$$f_z = Be^{\frac{2bz}{c}}, z > D, \quad (13)$$

for some constant B , while for $0 \leq z < D$ we will have:

$$f_z = C(e^{\frac{2bz}{c}} - 1), 0 \leq z < D, \quad (14)$$

because $f_0 = 0$. To insure continuity of f_z at $z = D$ we have:

$$Be^{\frac{2bD}{c}} = C(e^{\frac{2bD}{c}} - 1), \quad \text{or} \quad (15)$$

$$B = C(1 - e^{\frac{-2bD}{c}}). \quad (16)$$

From (12) in steady-state we set:

$$P = \lim_{z \rightarrow 0^+} [-bf_z + \frac{1}{2}c\frac{\partial f_z}{\partial z}] \quad (17)$$

and obtain:

$$P = bC \quad \text{or} \quad C = \frac{P}{b}. \quad (18)$$

Finally, setting $\gamma = \frac{2b}{c}$ and using (7) we obtain:

$$\begin{aligned} 1 &= P + \frac{P}{b} [\frac{1}{\gamma}e^{\gamma z}|_0^D - D + (1 - e^{-\gamma D})\frac{1}{\gamma}e^{\gamma z}|_D^{\infty}], \\ &= P - P\frac{D}{b}. \end{aligned} \quad (19)$$

As a consequence we have the following obvious intuitively result:

Result 1 If there is no packet loss and there are no time-outs, so that the packet is allowed to travel to its destination indefinitely until it gets there, the average time it takes the packet to reach its destination provided $b < 0$ is:

$$E[T] = \frac{D}{-b} \quad (20)$$

This formula, which only applies to the simplest case being considered, assures us about the feasibility of a search: as long as $b < 0$ then the average search time will

be finite. In this case, the second moment c of the distance traversed by the packet per unit time does not influence the average travel time.

However, in the sequel we will see that if packets can be lost, or destroyed (for instance due to a time-out), and are then re-transmitted from the source after some delay, then the average time to reach the destination will depend in a significant manner on the second moment of the diffusion process.

4. SEARCHING IN THE PRESENCE OF LOSSES

Packet loss can occur in many wireless or wired communication systems due to a variety of reasons, such as noise on the lines, the unavailability of a relay to forward the packet, congestion, etc. For some constant $\lambda \geq 0$ let $\lambda\Delta t$ be the probability that the packet is lost in any small enough time interval $[t, t + \Delta t[$.

It is rather common that the source will incorporate a time-out mechanism to retransmit the packet if it has not received an acknowledgement from the receiver by a certain time. Also, a packet may self-destroy if it has not successfully reached its destination by a certain time or after a certain number of hops.

Here we assume that the packet uses a time-out of length τ , when the packet destroys itself. Typically the time-out τ will have some constant value, but in order to make the model tractable we assume that it is an exponentially distributed random variable with parameter $r \geq 0$ and its average value is $E[\tau] = r^{-1}$.

After the time-out τ interval, the source decides to re-transmit the packet but in order to avoid retransmitting packets that may have actually reached their destination, we assume that an additional delay M elapses before the source actually sends a duplicate packet out. Of course, if $M = 0$ this simply means that the retransmission is effected instantaneously after the time-out occurs. We assume that M is exponentially distributed and that its average value is $E[M] = \mu^{-1}$. Note that the role of the time M would also be to avoid that the source of the packet should retransmit a packet too soon after the time-out is invoked; indeed, by introducing an additional delay M we are trying to limit spurious retransmission of packets that may have successfully made it to the destination without the source having been informed of the success.

In the section we evaluate the impact of possible packet losses, of the time-out delay, and of the overhead delay, on the total time it takes a packet to *finally* reach its destination, possibly after it has been retransmitted several times by the source.

We will assume that the loss or time-out of the packet can occur at any distance z from the destination, except obviously at the destination itself where $z = 0$. Note that the distance $z = D$ does not mean that the packet is at the source, because the packet may meander back to a distance equal to or larger than its initial distance to the destination.

As in the previous section, we represent the “end point” of the packet travel time by a probability mass $P(t)$ at point $z = 0$, so that $P(t)$ is the probability that the packet is at its destination at time t . We again use an artificial residence time at the end point of average value 1, after which the process representing the packet travel time repeats itself, and this process is repeated indefinitely. As a result, we will compute the steady-state probability P that the packet is at its destination point, and then obtain average time that it takes a packet to reach the destination

is given as before by $E[T]$ where:

$$\begin{aligned} P &= \frac{1}{1 + E[T]} \\ E[T] &= \frac{1}{P} - 1, \end{aligned} \quad (21)$$

so that the problem of calculating $E[T]$ reduces to the computation of P . In addition, we introduce:

- The probability $W(t)$ that the packet has been destroyed due to the time-out at some time prior to t , after which its retransmission will be re-initiated after a delay of average value $E[M] = \mu^{-1}$.
- The probability $L(t)$ that at time t the packet is lost; information about its loss will be available through the time-out only so that with rate $rL(t)$ the process enters the state $W(t)$.

Under these assumptions the equations governing the probability density function $f_{z,t}$ for the distance of the packet to its destination become:

$$\begin{aligned} \frac{\partial f_{z,t}}{\partial t} &= -(\lambda + r)f_{z,t} - b\frac{\partial f_{z,t}}{\partial z} + \frac{1}{2}c\frac{\partial^2 f_{z,t}}{\partial z^2} \\ &+ [P(t) + \mu W(t)]\delta(z - D), \end{aligned} \quad (22)$$

where the first term on the right hand side describes the effect of the loss of the packet and of its destruction due to the time-out, and:

- $P(t)$ multiplied by the rate 1 is the rate at which the packet travel process is re-started after the packet has successfully reached its destination, while
- $\mu W(t)$ is the rate at which the travel process is re-started after the overhead delay of average value μ^{-1} .

Hence:

$$\frac{dL(t)}{dt} = -rL(t) + \lambda \int_{0^+}^{\infty} f_{z,t} dz, \quad (23)$$

$$\frac{dW(t)}{dt} = -\mu W(t) + rL(t) + r \int_{0^+}^{\infty} f_{z,t} dz. \quad (24)$$

We also have that the sum of the probabilities is one:

$$1 = L(t) + W(t) + P(t) + \int_{0^+}^{\infty} f_{z,t} dz. \quad (25)$$

Finally we also have:

$$\frac{dP(t)}{dt} = -P(t) + \lim_{z \rightarrow 0^+} [-bf_{z,t} + \frac{1}{2}c\frac{\partial f_{z,t}}{\partial z}], \quad (26)$$

where the first term on the right-hand-side corresponds to the rate at which the search process is restarted after an “artificial rest period” of average time 1, while the second term represents the flow of probability mass towards the “arrival point” at $z = 0$. Again we have with the conditions that $f_{0,t} = 0$.

4.1 Solving the model in steady-state

We first write the diffusion equation in steady-state:

$$0 = -(\lambda + r)f_z - b\frac{\partial f_z}{\partial z} + \frac{1}{2}c\frac{\partial^2 f_z}{\partial z^2}, \quad (27)$$

and obtain the characteristic polynomial

$$0 = -(\lambda + r) - bu + \frac{1}{2}cu^2. \quad (28)$$

which has two real roots. Since we are seeking a function f_z which is a probability density whose integral over $[0, +\infty]$ must be finite, we take the negative root:

$$u = \frac{b - \sqrt{b^2 + 2c(\lambda + r)}}{c}. \quad (29)$$

so that:

$$f_z = Ae^{uz} + B, \quad 0 \leq z \leq D, \quad (30)$$

$$= Ce^{uz}, \quad z \geq D. \quad (31)$$

Using the condition $f_0 = 0$ we have $A = -B$ and with the continuity condition for f_z at $z = D$ we obtain $C = A(1 - e^{-uD})$. Thus after some calculations we get

$$\int_0^\infty f_z dz = -AD.$$

From (24) in steady state we obtain:

$$L = -\frac{\lambda}{r}AD, \quad (32)$$

$$W = -\frac{\lambda + r}{\mu}AD, \quad (33)$$

while (26) yields:

$$P = \frac{1}{2}cuA. \quad (34)$$

Using the normalising condition for the sum of the probabilities, we get:

$$A = \left[\frac{1}{2}cu - D\left(1 + \frac{r + \lambda}{\mu} + \frac{\lambda}{r}\right)\right]^{-1}, \quad (35)$$

and finally we have:

Result 2 The total average time that may include several possible restarts after time-outs and retransmission overhead delays, or losses with time-outs and overhead delays, is obtained by computing $E[T] = P^{-1} - 1$ using (34):

$$E[T] = -2D \frac{1 + \frac{\lambda+r}{\mu} + \frac{\lambda}{r}}{b - \sqrt{b^2 + 2c(\lambda + r)}}. \quad (36)$$

Note that if there are no packet losses ($\lambda = 0$), and if the time-out does not occur ($r = 0$), then we revert to the formula in *Result 1* as expected. Furthermore, if

$b > 0$ (the unfavorable case) and $b^2 \gg 2c(\lambda + r)$, then

$$E[T] \approx \frac{2Db}{c} \left(\frac{1}{r} + \frac{1}{\mu} \right). \quad (37)$$

Numerical Example Consider a numerical example obtained directly from the formula (36) that is relevant to the discussion we will have further on. Assuming that there are no losses, i.e. $\lambda = 0$, and $\mu = 0.1$ or $M = 10$ with $D = 10$, and $r = 0.02$ or $E[\tau] = 50$. With $c = 1$ and a positive drift parameter $b = 0.125$ we have $E[T] = 216.7$, while if we had $b = 0$ we would obtain $E[T] = 84.85$, showing how sensitive the model predictions are to the drift parameter b .

4.2 Optimising the Time-Out when b is negative

The case where $b \leq 0$ can be viewed as the “normal” or favourable situation when the packet typically makes some headway towards its destination each time it is transmitted. In this case, equation (4) can be written as:

$$E[T] = 2D \frac{1 + \frac{\lambda+r}{\mu} + \frac{\lambda}{r}}{|b| + \sqrt{b^2 + 2c(\lambda + r)}}, \quad (38)$$

and we see that as $r \rightarrow \infty$, $E[T] \rightarrow \infty$, and similarly as $r \rightarrow 0$ also $E[T] \rightarrow \infty$. Furthermore $E[T] > 0$ and it is continuous in r . Thus $E[T]$ will have a minimum value.

Figure 1 shows how the average time $E[T]$ is strongly influenced by the time-out and by the variance parameter c . We see that when packet loss can occur (in this case $\lambda = 0.1$), increasing variance parameter c actually reduces the average search time. This is because losses are followed by re-starts of the search, and a larger variance with the same average value increases the chance of finding a shorter path. We also see that there is definitely an optimum value of the time-out (in this case that minimises $E[T]$). The next Figure 2 illustrates the effect of the packet loss rate $\lambda = 0.1$ to $\lambda = 0.5$ for fixed c and varying r . It shows that as the loss rate increases, so does the average time $E[T]$ for the packet to reach its destination, as expected, if all other parameters remain the same. Again we see that the average value of the time-out will significantly influence the time it takes the packet to reach its destination and that an optimum value can be obtained.

4.3 Perfect Ignorance or b=0

A very interesting case, and somewhat surprising result, occurs for $b = 0$. This corresponds to the case where the packet is being routed under *perfect ignorance*: at each step it neither gets further away nor closer to the destination. We see that if packet loss can occur ($\lambda > 0$) or if time-outs exist ($r > 0$), the travel time of the packet to the destination remains finite on the average if $c > 0$ and is given by:

$$E[T] = 2D \frac{1 + \frac{\lambda+r}{\mu} + \frac{\lambda}{r}}{\sqrt{2c(\lambda + r)}}. \quad (39)$$

Thus even though we have a “stupid search” for the destination, without a clear direction which way to go, the packet does make it in a finite amount of time on

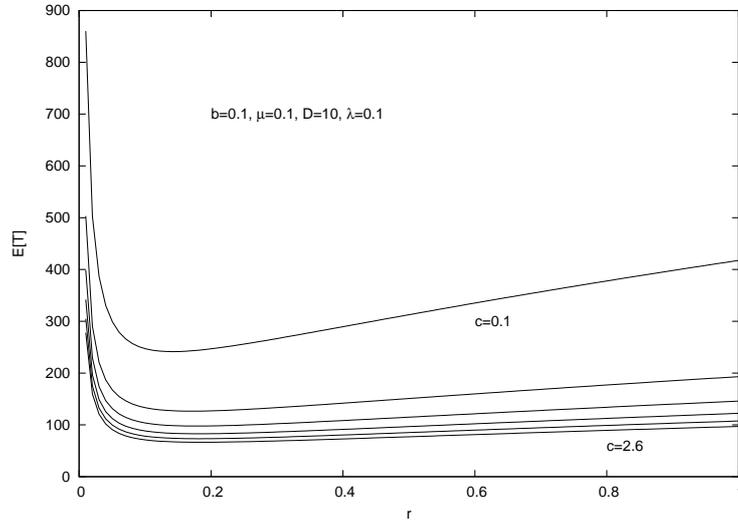


Fig. 1. The effect of varying the time-out rate r , for $D=10$, an average value of the overhead time for packet retransmission of 10, loss rate $\lambda = 0.1$ and different values of the variance parameter $c=0.1, 0.6, 1.1, 1.6, 2.1, 2.6$.

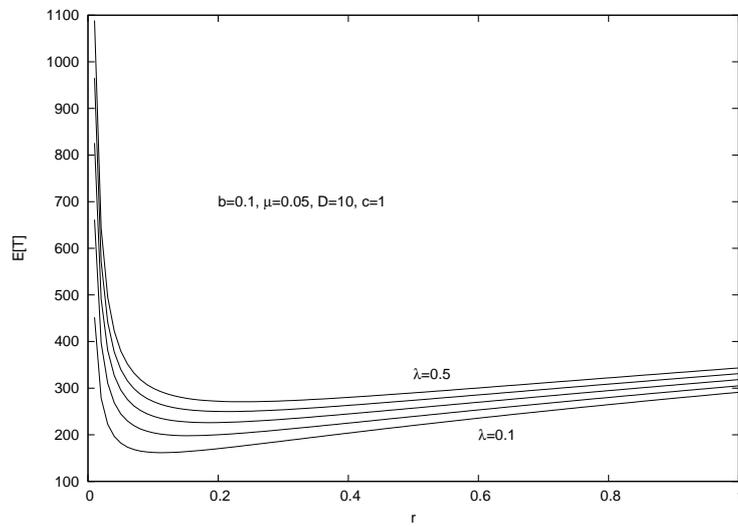


Fig. 2. The effect of varying the time-out rate r , with $b=0.1$, $D=10$, average value of the overhead before packet retransmission of 20, and values of the loss rate λ ranging from 0.1 to 0.5 in steps of 0.1.

the average. The numerical results of Figure 3 tell us that when the variance of the change in position per unit time c increases, the time to reach the destination decreases. The last Figure 4 shows how, for a fixed value of c , the loss rate influences

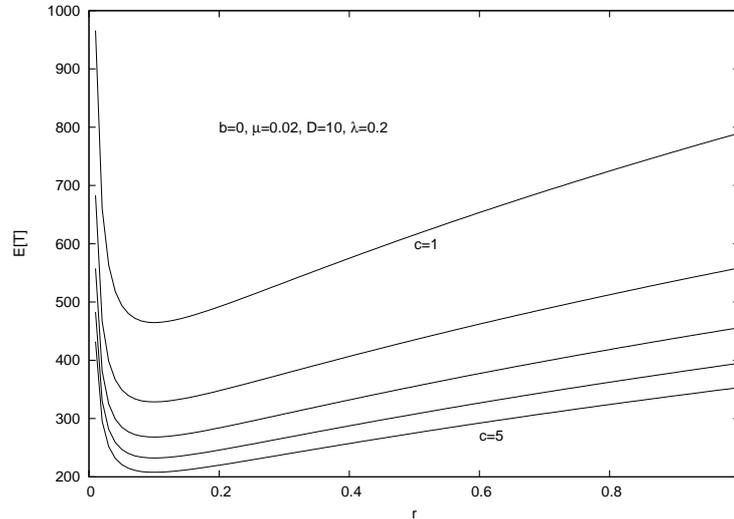


Fig. 3. The effect of varying the time-out rate r , with $b=0$, loss rate $\lambda = 0.2$, $D=10$, average value of the overhead before packet retransmission of 50, and values of the variance parameter c ranging from 1 to 5.

the average time $E[T]$. Again we see that there is an optimum value of the average time-out $1/r$ which results in the smallest value of $E[T]$.

5. SIMULATION EXAMPLES

In this section we will provide several simulation results to illustrate the preceding discussion. Each node in the network has a fixed wireless range d so that in order to go from some source to a destination, a packet will in general have to traverse multiple points. We assume that all nodes are reliable; in other words, if they are selected to receive a packet, they are able to do it, and are also able to retransmit the packet if needed at the next hop. We assume that there is no queuing in the nodes, so that packets which are being relayed by a node do not have to wait for a retransmission and the traversal of a node only takes a unit hop time. Furthermore, because the nodes are reliable the packets are never lost. In addition, there are no conflicts and collisions between packet transmissions at any of the nodes. However we do allow a node to discard a packet willfully via the “time-out”; the purpose is obviously be to destroy a packet that has been travelling in the network for too long. After the packet has visited τ hops and has still not reached its destination, then the packet is discarded. In the previous analysis, we had shown that this mechanism is essential for making sure a packet finally gets to destination when the direction it has to take is not known, or is imperfectly known.

In practice, the source may eventually know that the packet has been destroyed or lost. As an example, we can suppose that if K is some quantity that is significantly

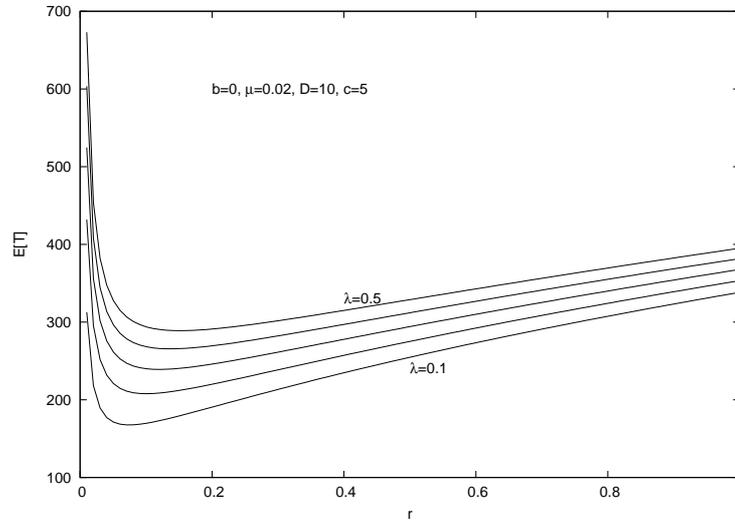


Fig. 4. The effect of varying the time-out rate r , with $b=0$, $D=10$, average value of the overhead before packet retransmission of 50, and loss rate values 0.1, 0.2, 0.3, 0.4, 0.5 .

larger than the estimated round-trip source-destination delay, then K time units after the packet has been sent out either the packet has reached its destination and an acknowledgement has returned to the source, or the packet must have been destroyed. If by that time the source has not received an acknowledgement, then it can conclude that the packet was indeed destroyed or lost and the source can proceed to send out a new copy of the packet. Notice that in the wireless case that we consider, if the packet does get to destination in some time say V , the acknowledgement packet can get back to the source much faster than $2V$ because there will now be a good estimate of the return path from the information that was gathered by the packet on the forward path. For instance, if the packet carries the geographic location of the source, then the destination can use this information to select a good path back to the source that eliminates needless random search.

Note that in the mathematical models of the previous sections, we have modelled the times τ and M by random exponentially distributed times of average values $E[\tau] = r^{-1}$ and $M = \mu^{-1}$, where $r\Delta t$ is the probability that a packet “timed-out” and placed in the “lost state” waiting for retransmission, and $\mu\Delta t$ is the probability that after it is considered lost it is retransmitted in a time interval of length Δt . In our simulations we have taken both of these quantities to be constants.

In the simulations, the wireless network nodes are placed at regular unit intervals in the infinite $i - j$ plane, i.e., there is a node that is placed at each point (i, j) where i and j are positive or negative integers. Packets are transmitted from a source, which is a given point in the grid, to some fixed destination point on the grid. We will simulate a “bad” scenario where any node (i, j) (i.e. either the source node or a relay node) has no knowledge of the “right direction” to take towards the destination, and it will then select at random *any one* of its reachable neighbours with equal probability and forward the packet to it. Concerning the range d , we

will consider two cases.

We first assume that the range is exactly $d = 1$, so that a node (i, j) can communicate with exactly four neighbours $(i \pm 1, j)$, and $(i, j \pm 1)$. As mentioned earlier, the time needed to send a packet from a node to any of its neighbours which are within range is $\Delta t = 1$. Thus the distance D between two nodes (i, j) and (l, k) is the minimum number of hops that separates them, or simply $D = |i - l| + |j - k|$. At some step of the forwarding of the packet, let the current node be (i, j) and the destination node be (l, k) . Two cases need to be considered:

- If the current distance of the node to its destination is $\delta > 1$, and $l \neq i$ and $k \neq j$, then the new distance will be $\delta - 1$ or $\delta + 1$ each with probability $1/2$.
- On the other hand, if either $l = i$ or $k = j$, then we see that the new distance will be $\delta + 1$ with probability $3/4$ and $\delta - 1$ with probability $1/4$. We will call this the “aligned” case, while the previous one will be called the “unaligned case”.

With the equal probability of 0.25 that a packet transmitted by a node is picked up by any of its neighbours, in the aligned case we will have that $b = +0.5$, so that there is a positive drift pushing the packet away from its destination. On the other hand, in the unaligned case we have $b = 0$. In both cases $c = 1$. In the first unaligned case $b = 0$, and in the aligned case $b = +1/2$. Thus we see that the value of b depends on the relative positions of the intermediate and the destination node. This leads us to consider how frequently a node may be aligned or unaligned with respect to the destination node, and this can be estimated from knowledge of δ the distance between the two nodes. Notice first that if $\delta = 1$, then the two nodes are always aligned so that $b = 0.5$.

More generally for any destination node and any value of δ there will be a total number of $(\delta + 1 - 2) \cdot 4$ unaligned nodes and 4 aligned nodes, so that although the aligned case is very infrequent when the packet is far from the destination, as it gets closer it is much more likely to occur. We can therefore estimate the average value of b over all nodes at distance δ from the destination to be:

$$\hat{b}(\delta) = \frac{1}{2\delta} \quad (40)$$

so that as the packet gets *closer* to the destination it gets more likely to *drift away* from it. Since the diffusion model we use does not include a dependence of the drift b on the distance z , it is “optimistic” with respect to the simulation results that we present.

In all the graphs of simulation results presented in this section, each point on a curves is the *average total packet travel time from source to destination*, averaged over *twenty simulation runs* with identical parameters. The simulation results for the case $d = 1$ case are shown in Figure 5 where we report the average *total* time it takes the packet to get to the destination (after multiple time-outs and retransmissions) versus the constant (not random) value of τ . Here we also take a constant value $M = 10$ so that after τ time units (hop times) the packet is discarded, and after $\tau + 10$ time units the packet is retransmitted if the packet had been discarded.

In the second case that we simulate the wireless range is $d = \sqrt{2}$ units so that a node (i, j) can communicate with exactly *eight* neighbours $(i \pm 1, j)$, $(i, j \pm 1)$, $(i +$

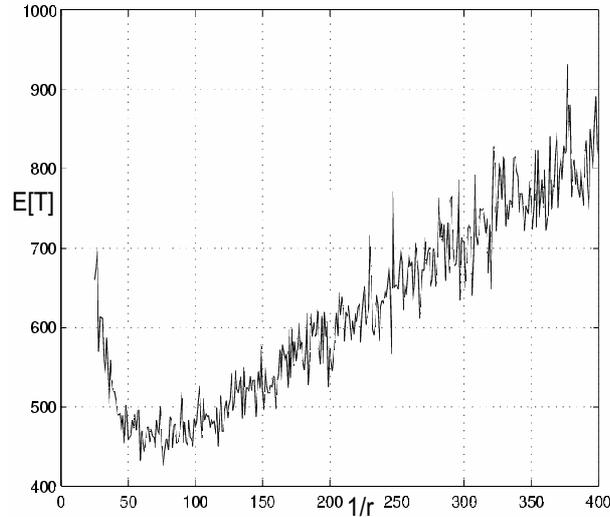


Fig. 5. The average time to the destination $E[T]$ (y-axis) versus the deterministic time-out value $\tau = r^{-1}$ (x-axis), obtained by simulation of packet transmission over the regular grid with $d = 1$ and four reachable neighbours to each node. We show the effect of varying the constant time-out τ , with no losses, source to destination distance $D = 10$, and a constant value of the wait time before packet retransmission of $M = 10$. We have $b = 0$ and the variance parameter is $c = 1$. Each point on the curve is the average of 20 simulation runs with the same parameter set.

$1, j-1$), $(i-1, j+1)$, $(i+1, j+1)$ and $(i-1, j-1)$. There are 8 reachable neighbours for any node, and we assume that the probability that a packet is accepted by any one of the reachable neighbours is 0.125. We can see that the average drift will be $b = 0$ for both aligned and unaligned nodes if they are far enough from the destination, and there will now be two neighbours (left and right node) for any node where the change in distance to the destination will be zero, so that we obtain $c = 0.75$. However when the distance is 1 we have $b = +0.25$. The results reported in Figure 5 are similar to those in the previous figure. However, as expected, the fact that each node can communicate with more neighbours means that the average travel times are all shorter. The analytical results shown in Figure 7 are again optimistic for all the reasons given previously, including the fact that as the packet gets closer to the destination it will tend to drift away, as explained above, is a significant reason for this difference. There can also be several other reasons: (1) the simulations use constant time-outs and overhead delays, rather than the exponentially distributed values of the theoretical model; (2) secondly, the simulations only use twenty samples for each point, which is small, however

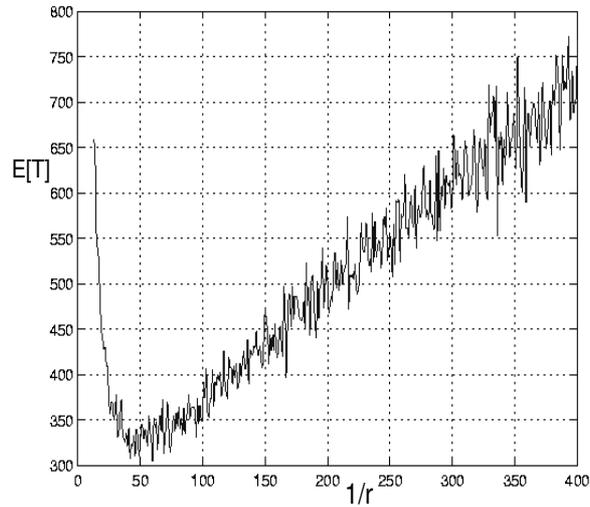


Fig. 6. Average time to destination $E[T]$ (y-axis) versus the deterministic time-out value $\tau = r^{-1}$ (x-axis), obtained by simulation of packet transmission over the regular grid with $d = \sqrt{2}$ and eight reachable neighbours to each node. We show the effect of varying the constant time-out τ , with no losses, source to destination distance $D = 10$, and a constant value of the wait time before packet retransmission of $M = 10$. We have $b = 0$ and the variance parameter is $c = 0.75$. Each point on the curve is the average of 20 simulation runs with the same parameter set.

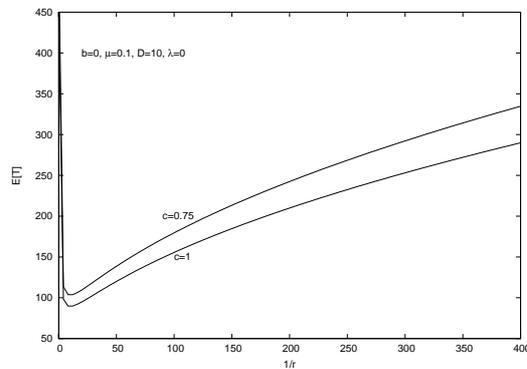


Fig. 7. Numerical results from the mathematical model. We show the effect of varying the average value of the exponentially distributed time-out $E[\tau] = r^{-1}$, with no losses, source to destination distance $D = 10$, and an exponential value of the wait time before packet retransmission of $M = 10$. We have $b = 0$, and the variance parameter is $c = 1$ for the case $d = 1$, and $c = 0.75$ for the case $d = \sqrt{2}$.

longer simulations would be impractical due to the computational time involved; (3) a distance of $D = 10$ is small for the continuous diffusion approximation to be accurate, but taking larger values such as $D > 100$, would make the simulations impractically long to run; (4) the simulations will have a drift parameter which will push the packet away from the destination as it gets closer, as discussed earlier, while the mathematical model assumes that $b = 0$ and that it does not depend on the distance z .

6. CONCLUSIONS

In this paper, closed form analytical results are derived for the average travel time of a packet in a homogenous multi-hop medium as a function of the distance between the source and the destination, using a continuous space approximation. We consider the case where time-outs are used to destroy packets that have been in the network too long without reaching their destination, and are then replaced with fresh packets, as well as the simpler case where time-outs are not used. We show that in all cases, an optimum value of the time-out interval exists that will minimise the average travel time. We show how the average travel time will depend on the packet loss rate, the overhead delay for retransmission after a time-out, as well as the average progress rate of a packet and the variance of the progress of the packet towards its destination.

We demonstrate, both by simulations and analytically, that even when the packet *does not* on the average make progress towards its destination at each step, the time-out guarantees that the packet eventually makes it to the destination in a time which is finite on average provided that the packet's behaviour is random, i.e. that it does not always make the "same mistakes".

The approach we have taken can be extended in several directions. In addition to the total travel time T , other quantities of interest could be computed such as the probability that the packet is within an ϵ - *neighbourhood* of its destination. One could also study how the time-out interval, and the loss probability, can depend on the fact that the packet has gotten too far away from its destination. It will also be interesting to consider models where the destination node moves, so as to evaluate the impact of node mobility on packet travel time.

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