Packet Transmission with K Energy Packets in an Energy Harvesting Sensor

Yasin Murat Kadioglu
Intelligent Systems and Networks Group
Dept. of Electrical and Electronic Engineering
Imperial College, London SW7 2BT, UK
y.kadioglu14@imperial.ac.uk

Erol Gelenbe
Intelligent Systems and Networks Group
Dept. of Electrical and Electronic Engineering
Imperial College, London SW7 2BT, UK
e.gelenbe@imperial.ac.uk

ABSTRACT

This paper discusses a two-dimensional random walk for modeling a data transmission system with energy harvesting that represents a remotely operating wireless sensor node. The node has a wireless transmitter that gathers data from the environment according to a random process, and similarly it harvests energy from the environment at a random rate. Then if it has gathered enough data, it transmits a data packet provided it has stored at least $K \geq 1$ energy packets that are needed to transmit one data packet. Generalising previous results that were obtained for the case $K = 1$, we derive the stationary probability distribution of this model to compute principal performance metrics for the wireless sensor node with energy harvesting, including the average transmission power emanating from any sensor node. We then compute the probability that a bit is correctly received by a receiver that operates in the presence of N identical wireless sensors, each operating at the power level K, in the presence of noise and of interference due to the transmissions. Numerical results show how the noise power B and the transmission power K affect the probability of correctly receiving a bit in such a system.

Keywords

Wireless Sensors; Energy Harvesting; Energy Packets; Data Packets; Data Transmission; Markov Chain

1. INTRODUCTION

A wireless sensor node is a tiny device that can sense changes in the environment including temporal differences in chemical, thermal, or magnetic conditions, and which is capable of performing some simple processing and forwarding the data to other connected nodes or to a sink [2]. Such devices can be used for several purposes in environmental, health, energy, home or security applications [16, 4, 12], with minimal human intervention and no direct connection to the power mains, but with the disadvantages of finite battery capacity and battery aging [15, 14].

Thus they should preferably be autonomous without the need of frequent battery changes, and able to harvest energy from the environment by using energy harvesting [17] from mechanical vibrations, light, heat, or radiation [3, 1, 18].

Earlier work [7] introduced the idea that such systems could be studied in a discretised manner, where both the data packets (DPs) and the energy that is stored in a battery or capacitor can be presented as discrete units where one or more "energy packets" (EPs) are used to correctly transmit a single DP in the presence of transmission errors [10]. In another related but distinct approach, work and energy flows in a computer system [6] are modeled using queuing network models known as G-networks [5], as described in recent paper on the optimisation of computer and communication systems that use intermittent sources of energy [9].

In this paper we generalise the models in [7, 8] to the case where exactly K energy packets are needed for a successful transmission when there is no transmission errors (no interference and noise), and $K \geq 1$. The motivation lies in the fact based on noise or interference, the transmitter may need to make use of a higher transmission power level, or it may wish to reduce it to save energy.

We make the simplifying assumption that data gathering, i.e. sensing itself, does not deplete the stored energy. This actually means that the sensing process, including analog to digital conversion and data storage in a data buffer, may benefit from a separate energy harvesting mechanism and that we are just analysing the energy used for data transmission. We also assume that energy leakage is negligible as compared to the energy used for transmission. Another basic assumption that we carry over from prior work is that the actual transmission times of packets, which may be in the nanoseconds, is negligible as compared to the longer times needed to gather sensory DPs and to harvest EPs. In the analysis we present, the the number of DPs that the sensor can store simultaneously cannot exceed B, while the battery or capacitor has a maximum energy storage capacity of E EPs. In the sequel, the system that we have described is modeled as a two-dimensional random walk, and its principal performance characteristics are computed in steady-state.

2. THE MATHEMATICAL MODEL

The wireless sensor node that we model receives data and energy from the environment in a random manner. The arrival of DPs and EPs occur according to two distinct inde-

Energy-Sim, June 21-24, 2016, Waterloo, ON, Canada

DOI: http://dx.doi.org/10.1145/2939948.2939949

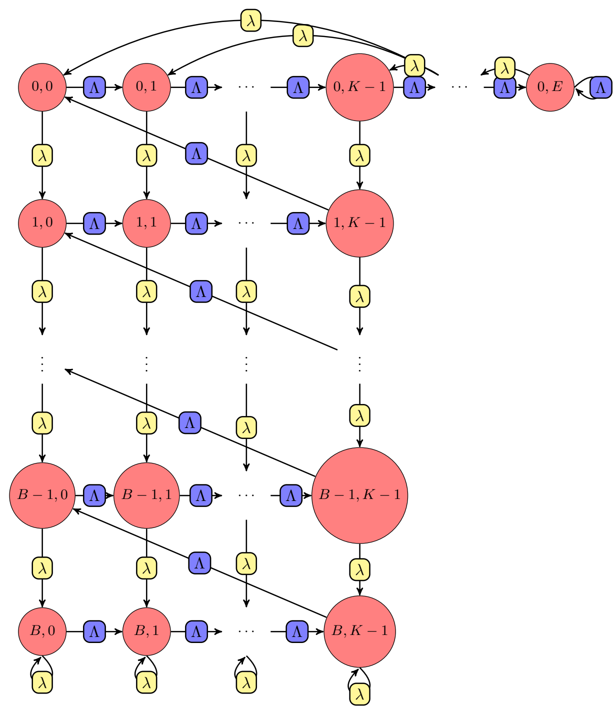

Figure 1: Markov chain state diagram

pendent Poisson processes with rates λ and Λ, respectively. DPs are stored in a data buffer and EPs are stored in an energy buffer (capacitor or battery) at the node. We assume that the energy and data arrival rates to the node are very small, but the time it takes to create packets and transmit them is extremely short compared to these rates so that the transmission time is negligible, i.e. transmission of a DP is instantaneous and takes zero time.

The state of the sensor node is represented by the random process $(N(t), M(t))$ where $N(t)$ is the number of data packets stored at the node while $M(t)$ is the number of energy packets that are stored, both at time $t \geq 0$. Because of the very small processing and transmission times at the node, whenever enough energy packet is available and there are data packets waiting they will be instantaneously transmitted till the energy or data packets are depleted.

Let us write $p(n, m, t) = Prob[N(t) = n, M(t) = m]$. From the above remark, we need only consider $p(n, m, t)$ for the state space S of pairs of integers $(n, m) \in S$ such that: $S = \{(0, 0), (n, 0), (0, m), (l, k)\}$ such that $1 \leq n \leq B, 1 \leq m \leq E, 1 \leq l < B, 1 \leq k < K$ where B (E) is the maximum amount of DP (EP) can be stored in the data (energy) buffer. According to transmission scheme defined, we can observe the state diagram in Figure 1 and stationary probability distribution for each of these states can be computed from following balance equations:

$$p(0, 0)[\lambda + \Lambda] = \Lambda\ p(1, K-1) + \lambda\ p(0, K) \tag{1}$$

$$p(n, 0)[\lambda + \Lambda] = \Lambda\ p(n+1, K-1) + \lambda\ p(n-1, 0) \tag{2}$$
$$\mathbf{1 \leq n < B}$$

$$p(B, 0)[\Lambda] = \lambda\ p(B-1, 0) \tag{3}$$

$$p(0, m)[\lambda + \Lambda] = \tag{4}$$
$$\Lambda\ p(0, m-1) + \lambda\ p(0, m+K) 1[E \geq m+K]$$
$$\mathbf{1 \leq m < E}$$

$$p(0, E)[\lambda] = \Lambda\ p(0, E-1) \tag{5}$$

$$p(l, k)[\lambda + \Lambda] = \Lambda\ p(l, k-1) + \lambda\ p(l-1, k) \tag{6}$$
$$\mathbf{1 \leq l \leq B-1, 1 \leq k \leq K-1}$$

$$p(B, k)[\Lambda] = \Lambda\ p(B, k-1) + \lambda\ p(B-1, k) \tag{7}$$
$$\mathbf{1 \leq k \leq K-1}$$

Finding a closed-form expressions for the stationary probability distributions is elusive by considering above expressions. However, we can find them by modifying the system. We can define a *one-to-one* and *onto* function that combines data and energy indices of the states into a single index such that $p(n, m) = \tilde{p}(nK - m + E)$. When we consider this particular function, we can observe that there is a decreasing order among the states starting from the $(BK + E)$ to (0) with increment 1 so that this diagram can be modeled as 1D Markov chain which can be seen in Figure 2. For the reduced Markov chain model, transitions among states can be separated basically in 3 different regions in order to reduce complication of the analysis. Thus, we can write balance equations as follows:

- *Region1,* $BK + E - K < N \leq BK + E$*:*

$$\tilde{p}(BK + E)\Lambda = \lambda \tilde{p}(BK + E - K), \tag{8}$$
$$\tilde{p}(N)\Lambda = \lambda \tilde{p}(N - K) + \Lambda \tilde{p}(N + 1). \tag{9}$$

- *Region2,* $K \leq N \leq BK + E - K$*:*

$$\tilde{p}(N)[\Lambda + \lambda] = \lambda \tilde{p}(N - K) + \Lambda \tilde{p}(N + 1). \tag{10}$$

- *Region3,* $0 < N < K$ *:*

$$\tilde{p}(N)[\Lambda + \lambda] = \Lambda \tilde{p}(N + 1), \tag{11}$$
$$\tilde{p}(0)\lambda = \Lambda \tilde{p}(1). \tag{12}$$

The solution of (9) leads us to find the stationary probabilities in Region 2, and it is a recurrence relation of order of $(k + 1)$ whose characteristic equation is :

$$\Theta^{K+1} - (1 + \frac{\lambda}{\Lambda})\Theta^K + \frac{\lambda}{\Lambda} = 0. \tag{13}$$

Equation (12) has $K + 1$ roots, namely $\{\vartheta_1, \vartheta_2, ..., \vartheta_{K+1}\}$ so that the closed-form expression for the distribution is:

$$\tilde{p}(N) = c\Theta^N = c(\sum_{j=1}^{K+1} a_j \vartheta_j)^N \tag{14}$$

where c and a_js are arbitrary constants.

By applying Descartes' rule of signs, we can conclude that (12) has two or zero positive real roots, one negative real root and its other roots are complex valued. Here, we can rewrite (12) as

$$\Theta^K (\lambda + \Lambda(1 - \Theta)) - \lambda = 0. \tag{15}$$

Since 1 is a root of this equation, (12) has exactly 2 positive real roots.

The set of $a_j's$ should such that Θ must be real valued in the interval $(0, 1)$. When we consider the state $(K - 1)$, by

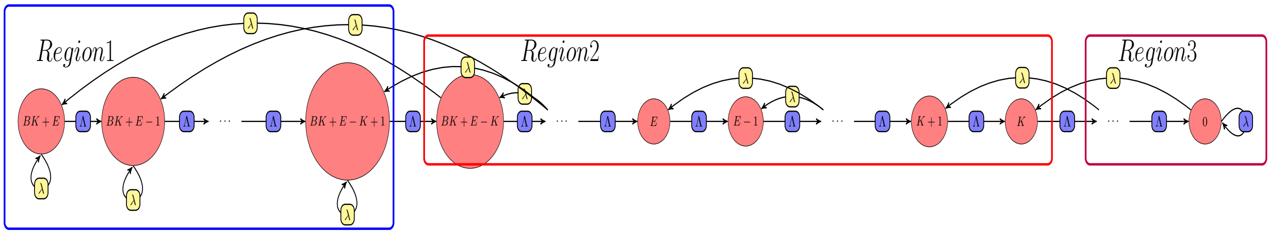

Figure 2: 1D Markov chain representation for states with single index

(10) we can write:

$$\tilde{p}(K-1) = \frac{\Lambda}{\Lambda+\lambda} c\Theta^K. \tag{16}$$

By using (15) in further calculations, we can express stationary probabilities in Region 3 as follows:

$$\tilde{p}(K-i) = (\frac{\Lambda}{\Lambda+\lambda})^i c\Theta^K,\ 0 < i < K \tag{17}$$

and

$$\tilde{p}(0) = \frac{\Lambda}{\lambda}(\frac{\Lambda}{\lambda+\Lambda})^{K-1} c\Theta^K. \tag{18}$$

Also, when we consider the state $(\xi+1)$ where $\xi = (BK+E-K)$, by (9) we can write:

$$\tilde{p}(\xi+1) = c\Theta^\xi(1+\frac{\lambda}{\Lambda}(1-\Theta^{-K})). \tag{19}$$

By using (18) in further calculations, we can express stationary probabilities in Region 1 as follows:

$$\tilde{p}(\xi+m) = c\Theta^\xi\left(1+\frac{\lambda}{\Lambda}(1-\Theta^{-K}\sum_{i=0}^{m-1}\Theta^i)\right), \mathbf{0 < m < K}$$

and

$$\tilde{p}(\xi+K) = \frac{\lambda}{\Lambda} c\Theta^\xi.$$

Using the fact that the sum of the probabilities is 1:

$$\sum_{N=0}^{BK+E}\tilde{p}(N) = \sum_{N=0}^{K-1}\tilde{p}(N) + \sum_{N=K}^{\xi} c\Theta^N + \sum_{N=\xi+1}^{BK+E}\tilde{p}(N) = 1.$$

If $0 < \Theta < 1$, summation reduces :

$$c\Theta^K\frac{\Lambda}{\lambda} + \frac{c(\Theta^K - \Theta^{\xi+1})}{1-\Theta} + \sum_{m=1}^{K}\tilde{p}(\xi+m) = 1. \tag{20}$$

where

$$\sum_{m=1}^{K}\tilde{p}(\xi+m) = \tag{21}$$

$$c\sum_{m=1}^{K-1}\left(\Theta^\xi + \Theta^\xi\frac{\lambda}{\Lambda} - \Theta^{\xi-K}\sum_{i=0}^{m-1}\Theta^i\right) + c\Theta^\xi\frac{\lambda}{\Lambda}.$$

After further algebra, (20) reduces:

$$c\Theta^\xi(K(1+\frac{\lambda}{\Lambda})-1) + c\Theta^{\xi-K}\left(\frac{\Theta^K - K(\Theta-1)-1}{(\Theta-1)^2}\right).$$

Thus:

$$\sum_{N=0}^{BK+E}\tilde{p}(N) = \tag{22}$$

$$c\Theta^K\frac{\Lambda}{\lambda} + \frac{c(\Theta^K - \Theta^{\xi+1})}{1-\Theta} +$$

$$c\Theta^\xi[K(1+\frac{\lambda}{\Lambda}) - 1 - \frac{\Theta^{-K}(1+K(\Theta-1))-1}{(\Theta-1)^2}]$$

$$= 1.$$

Also, for the sake of simplicity, we can assume an infinite data buffer which overcomes the different balance equations between Region 1 and Region 2 so that (21) reduces to:

$$\sum_{N=0}^{\infty}\tilde{p}(N) = c\Theta^K\left(\frac{\Lambda}{\lambda} + \frac{1}{1-\Theta}\right) = 1. \tag{23}$$

After some further computations, we obtain:

$$\Theta^{K+1} - \left(1+\frac{\lambda}{\Lambda}\right)\Theta^K + \frac{\lambda}{c\Lambda} - \frac{\lambda\Theta}{c\Lambda} = 0, \tag{24}$$

and when we substitute (12) into (23), we get :

$$c = 1 - \Theta. \tag{25}$$

Thus, the solution is :

$$\tilde{p}(N) = \begin{cases} (1-\Theta)\Theta^N, & K \le N < \infty \\ (1-\Theta)\Theta^K(\frac{\Lambda}{\Lambda+\lambda})^i, & N = K-i,\ 0 < i < K \\ (1-\Theta)\Theta^K\frac{\Lambda}{\lambda}(\frac{\Lambda}{\lambda+\Lambda})^{K-1}, & N = 0 \end{cases}$$

where Θ is the summation of linearly combined roots of equation (12). Note that (12) cannot be solved in radicals for $K \ge 4$ by the Abel & Ruffini theorem [19], which means that there does not exist an expression for the roots of such equations as a function of the coefficients by means of algebraic operations and roots of natural degrees. Thus, it is better to adjust the system model that one data packet can be transmitted by 4 or less energy packet.

3. MODELING INTERFERENCE AND NOISE

As previously, in this section the arrival rate of energy packets Λ is itself in units of power, i.e. the flow of energy packets per second maps into power entering the wireless node via harvesting. Assume that a single energy packet contains a unit of energy and that the power level that has been set to transmit one data packet is K.

On the other hand, *on average* the total radiated power ϕ from a node is simply the power entering the node from harvesting, minus that which is lost through energy loss due to the finite capacity of the energy buffer, so that we are assuming a "perfect" transmitter that does not waste any power in electronics. Of course this is unreasonable, but the lost effect may be "hidden" in the value Λ, i.e. this rate is merely the amount of power that reaches the transmitter after the energy is harvested and then used (in part) to operate the sensor's electronic circuits. We then have:

$$\phi = \Lambda - L_e,$$

where

$$L_e = \Lambda \sum_{n=0}^{\infty} p(n, E) = \Lambda p(0, E) = \Lambda \tilde{p}(0).$$

Now consider a particular sensor, say the i-th, operating in proximity with a total of N wireless sensors all transmitting at the same power level K. In the communication channel due to noise of power level B at the receiver, plus the interference at level I from the other sensors, assuming that the transmission of a $+1$ is as likely as that of the transmission of a -1, we can write the probability C of correct detection of a bit by the receiver as:

$$C = \frac{1}{2}\{Prob[\alpha K > \alpha I + B] + Prob[-\alpha K + \alpha I + B \leq 0]\}, \quad (26)$$

where $0 \leq \alpha \leq 1$ is the fraction of transmitter power received with respect to the amount transmitted, and $I = \sigma\phi(N-1)$ where $0 \leq \sigma \leq 1$ represents the "side-band interference" effect; indeed if multiple transmitters use some closely related frequency, they will avoid using exactly the same frequencies, but their sidebands will interfere with each other so that we may expect σ to be much less than one. With BPSK transmissions where both the interference and the noise are assumed to be Gaussian of zero mean [13] the probability of correctly receiving a binary symbol is then:

$$C = 1 - Q(\sqrt{\frac{\alpha K}{\alpha\sigma\phi(N-1) + B}}), \quad (27)$$

where $Q(x) = \frac{1}{2}[1 - erf(\frac{x}{\sqrt{2}})]$.

The value of K has an interesting effect on the error rates. On the one hand, a bigger K can cause ϕ to grow, and hence it creates more interference, but it also provides higher power to overcome interference as well as noise. So this interesting effect will be illustrated below.

3.1 Numerical Examples

An increase in the total number of sensors N will potentially increase the interference in the communication system, so that C may decrease. We can observe this effect in Figure 3, where each data packet is a singe bit $\{-1, +1\}$, and $\alpha = 0.5$, $\sigma = 0.1$, $B = 0.1$, $\Lambda = 20$, $\lambda = 1$ for several values of K, and N varies between 2 and 30. Also, when the required energy to transmit one bit is increased, i.e. K is increased, we can observe that C is slightly higher for the same number of transmitters N.

We may also consider the effect of different noise power levels on C as illustrated in Figure 4. Taking the same parameters as for Figure 3, we set $N = 30$, and we can observe that C decreases nearly linearly as B increases, and that the effect of K is quite limited.

Figure 5 shows a new effect that we did not discuss previously. In real systems the total number of frequency bands

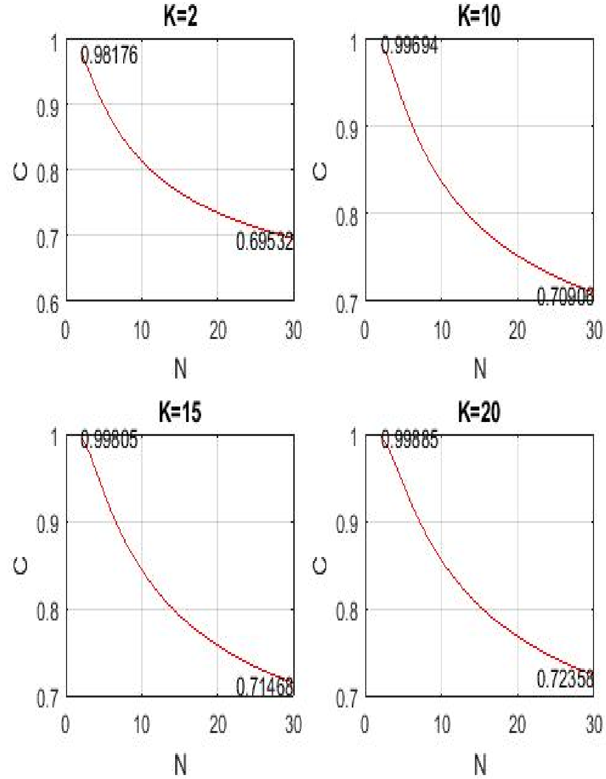

Figure 3: The relation between the probability of correct detection of a bit C and the number of simultaneously transmitting wireless sensors N, for different transmission power levels K.

which are being multiplexed will be limited; here that number is set to 30. Thus when we exceed $N = 30$ the level of interference is assumed to grow dramatically, passing from a $\sigma = 0.1$ to $\sigma = 1$, taking an extreme *worst case* case that all the other stations interfere with the one that we are analysing.

3.2 More Accurate Modeling of Interference

Though in this paper we do not wish to enter into great details regarding the communication channel, we would like to point to a more accurate manner of modeling a channel that uses N_0 distinct frequencies. When there are up to N_0 active transmitters, we would expect the interference at the receiver to take the form $I = \alpha\sigma_0\phi(N-1)$. However, more generally, we would expect that up to N_0 simultaneous transmissions, each transmitter will use a distinct frequency, while above that value the other transmissions will simply create an additional direct interference with one of the transmitters chosen at random so that we may have:

$$I = \sigma\alpha\phi(N-1) + 1[N > N_0].\alpha\phi\frac{N - N_0}{N}. \quad (28)$$

4. CONCLUSIONS AND FUTURE WORK

This paper analyses a wireless sensor node that gathers both data and energy from environment, so that they can operate autonomously. A stochastic model of the harvested

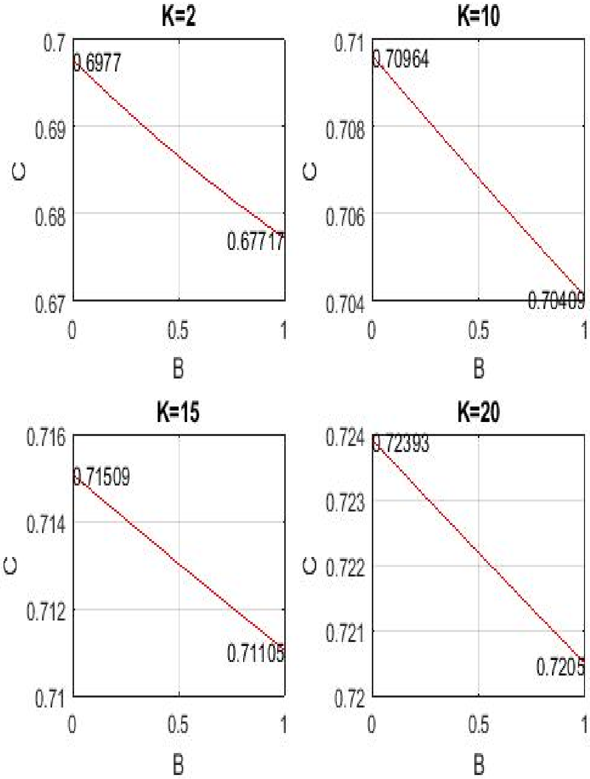

Figure 4: The relation between C and the noise power B for different values of transmission power K with $N = 30$ mutually interfering transmitters.

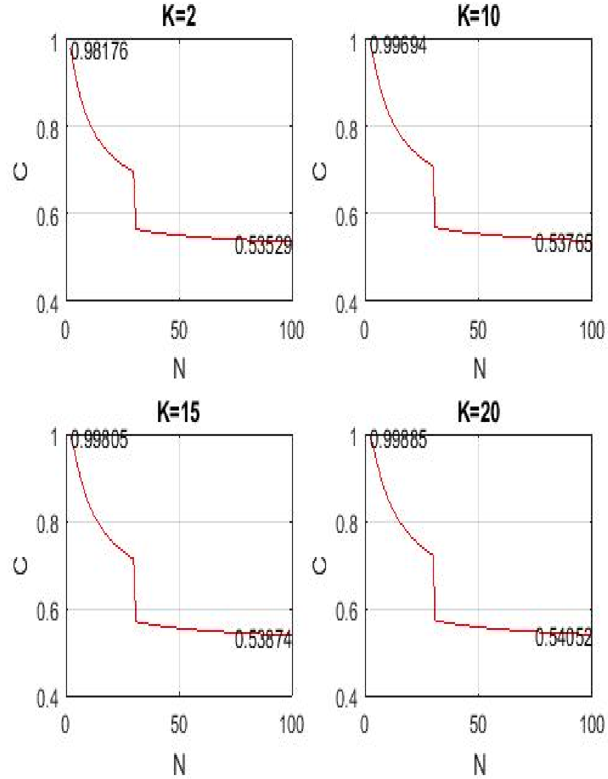

Figure 5: C versus the number of interfering transmitters as a function of N when we assume a much greater interference effect, represented by $\sigma = 1$, when N exceeds the number of multiplexed frequency bands assumed to be 30. Different transmission power levels K have little effect on the results.

energy and the data arrival is considered with a transmission scheme by which one data packet can be transmitted with the help of several energy packets. The system is analysed using a random walk model that represents the random arrivals of data packets, as well as the random arrivals of harvested energy in the form of discrete data packets. While data packets are stored in a buffer, energy packets are also stored in an "energy buffer or store" which represents a battery or capacitor.

Using precise assumptions about the random processes that are involved, we obtain the stationary distribution of buffer lengths with limited data and energy buffers, and both data packets and energy packets can be lost when their respective buffers are full. This analysis also allows us to compute the average transmitted power from a sensor, and also to study the behavior of one sensor in the presence of a collection of interfering sensors as well as of noise at the receiver. In particular, we can also compute the probability that a finite set of bits is correctly received by a receiver in the presence of several identical wireless sensor transmitters. Numerical examples are then used to illustrate the contradictory effect of the transmitter power: high power levels can improve the probability of correct packet reception, but they can also increase interference and have exactly the opposite effect.

Future work will address the processing energy consumption as well as the energy consumption for transmission. We also hope to generalise our results to multi-hop systems similar to the approach taken in [11].

Acknowledgements

We gratefully acknowledge the support of the ERA-NET ECROPS Project under EPSRC Grant No. $EP/K017330/1$ to Imperial College.

5. REFERENCES

[1] A. M. A. Rahimi, O. Zorlu and H. Külah. A fully self-powered electromagnetic energy harvesting system with highly efficient dual rail output. *IEEE Sensors Journal*, 12(6), June 2012.

[2] G. Anastasi, M. Conti, M. D. Francesco, and A. Passarella. Energy conservation in wireless sensor networks: A survey. *Ad Hoc Networks*, 7(3):537 – 568, 2009.

[3] M. A. Antepli, E. Uysal-Biyikoglu, and H. Erkal. Optimal packet scheduling on an energy harvesting broadcast link. *IEEE J. on Selected Areas in Comm.*, 29(8):1712–1731, 2011.

[4] A. Filippoupolitis, L. Hey, G. Loukas, E. Gelenbe, and S. Timotheou. Emergency response simulation using

wireless sensor networks. In *Ambi-Sys '08, Proceedings of the 1st International Conference on Ambient Media and Systems*, page 21. ICST (Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering), 2008.

[5] E. Gelenbe. The first decade of g-networks. *European Journal of Operational Research*, 126(2):231–232, 2000.

[6] E. Gelenbe. Energy packet networks: Ict based energy allocation and storage - (invited paper). In *GreeNets2011: First ICST International Conference on Green Communications and Networking*, pages 186–195, October 2011.

[7] E. Gelenbe. A sensor node with energy harvesting. *ACM SIGMETRICS Performance Evaluation Review*, 42(2):37–39, 2014.

[8] E. Gelenbe. Synchronising energy harvesting and data packets in a wireless sensor. *Energies*, 8(1):356–369, 2015.

[9] E. Gelenbe and E. T. Ceran. Energy packet networks with energy harvesting. *IEEE Access*, pages 1321–1331, 2016.

[10] E. Gelenbe and Y. M. Kadioglu. Energy loss through standby and leakage in energy harvesting wireless sensors. In *20th IEEE International Workshop on Computer Aided Modelling and Design of Communication Links and Networks (CAMAD)*, pages 231–236. IEEE, 2015.

[11] E. Gelenbe and A. Marin. Interconnected wireless sensors with energy harvesting. In *Proceedings of Analytical and Stochastic Modelling Techniques and Applications (ASMTA'15)*, pages 87–99. Springer International Publishing Switzerland, May 2015.

[12] E. Gelenbe and F.-J. Wu. Large scale simulation for human evacuation and rescue. *Computers & Mathematics with Applications*, 64(12):3869–3880, December 2012.

[13] A. Goldsmith. *Wireless communications*. Cambridge university press, 2005.

[14] M. Jongerden, A. Mereacre, H. Bohnenkamp, B. Haverkort, and J. P. Katoen. Computing optimal schedules of battery usage in embedded systems. *Industrial Informatics, IEEE Transactions on*, 6(3):276–286, 2010.

[15] M. R. Jongerden and B. R. Haverkort. Which battery model to use? *Software, IET*, 3(6):445–457, 2008.

[16] M.-S. Pan, C.-H. Tsai, and Y.-C. Tseng. Emergency guiding and monitoring applications in indoor 3D environments by wireless sensor networks. *International Journal of Sensor Networks*, 1(1/2):2–10, 2006.

[17] F. K. Shaikh and S. Zeadally. Energy harvesting in wireless sensor networks: A comprehensive review. *Renewable and Sustainable Energy Reviews*, 55:1041–1054, 2016.

[18] J. Yang and S. Ulukuş. Optimal packet scheduling in an energy harvesting communication system. *IEEE Trans. Commun.*, 60:220–230, 2012.

[19] H. Żołądek. The topological proof of abel–ruffini theorem. *Topol. Methods Nonlinear Anal*, 16:253–265, 2000.