ECROPS Final Report from Imperial College: Energy Packet Networks and Energy Harvesting

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Abstract

This report analyses the cooperation among energy prosumers (unified energy provider and consumer) through the Energy Packet Network paradigm which represents both the flow of work that requires energy, and the flow of energy itself, in terms of discrete units. We present two streams of work. The first approach presents a random walk model of a wireless communication node that exploits energy harvesting. The second part details a stochastic model of EPNs that is inspired from a branch of queueing theory called G-networks. Both approaches yield the equilibrium state of a system that includes energy storage units, energy transmission networks and energy consumers, together with the intermittent energy sources. The models are used to show how intermittent or renewable energy needs to balance the flow of communications and computer processing, and how the flow of work and energy in the system can be optimised utility functions that take into account both the needs of the consumers, and the desire to maintain reserve energy for future needs.

I. Introduction

In the future, billions of computer and communication devices may work through the Internet of Things (IoT) to manage and solve locally the major challenges of cities and human communities [1]. Applications will include energy provisioning and the smart grid [2], physical safety and security [3], health [4] and the environment [5]. In particular, the smart grid will itself require innovative communication solutions [6], [7], [8], [9], [10], [11], but may also raise synchronisation problems [12] between the information flows regarding the energy and workloads, raising interesting questions about how the smart grid can be effectively monitored and managed.

However such energy and workload monitoring and management systems will also consume energy and add to societal issues such as CO_2 impact, cybersecurity and reliability [13]. Thus in order to move forward towards ubiquitous, self-sustainable and optimised systems such as smart cities, sustainable agriculture, energy microgrids, and local energy supply systems, with desirable properties such as reduced emissions and energy transmission losses, it will be necessary to power the IoT as much as possible through sources of renewable and clean energy [14]. Similarly, it will also be necessary to meet the energy needs of the communication networks, servers and data centres that are needed to support them [15].

In the meanwhile, the worldwide increase in electrical energy consumption for ICT [16] worldwide in the order of 5-7% per year, has also motivated research in energy harvesting [17], [18], [14], [19] for computer and communications systems. Furthermore power is being increasingly generated in a distributed manner with consumers becoming "prosumers", who may also be producers and/or "storers" of energy (e.g. in home-based battery systems, or in individual vehicles). Thus there is an increasing motivation to dynamically manage electrical energy consumption, in conjunction with its generation and distribution. Such dynamic management requires the integration of consumers with communication networks and data centres so that energy management software may dynamically optimise the power grid.

These evolutions encourage us to study integrated models for renewable energy systems in the presence of dynamic and flexible energy consumption [20]. In this respect, a variety of emerging technologies such as Intelligent Power Switches, PowerComm Interfaces [21] and wireless energy transfer [22] can assist in dynamically managing, storing and conveying electrical energy. Furthermore, energy storage in conjunction with harvesting, energy sharing between prosumers, and workload migration between certain energy consumers such as data centres, can be used to improve the economic cost and CO_2 impact of the data centres and computer networks that are required to support the energy systems of the future. Since energy harvesting will in general be intermittent, it will useful to store energy dispatch it dynamically to optimise overall system behaviour [23], [24]. Thus this report addresses various facets of the the "Energy Packet Network" (EPN) paradigm [25], [26], [27] to model and optimise the dynamic generation, storage and consumption, as well as the dynamic workload, of highly dynamic and distributed energy systems.

We first focus on computer communication nodes whose energy needs are met by a combination of rechargeable storage batteries (or capacitors) fed by energy harvesting, together with a possible connection to the mains as a back-up when the harvesting cannot take place and the batteries are depleted. Much work in recent years has been devoted to energy savings and energy harvesting in wireless communications [28], [29], [30], [7]. Smart techniques have been suggested to adapt transmission to the energy and channel conditions so as to save energy and adapt the transmission power cooperatively so as to minimise the energy consumption per successfully transmitted packet [31], [32], [33], [34], [35], [36]. Specifically in this article we study a node that has a data storage backlog (*i.e.*, "data buffer") and has an energy storage device (*i.e.*, "energy buffer").

The arrival of energy and of data packets to the nodes are both random processes: energy flows in at random through energy harvesting. Data accumulates into the node, also at random, through sensing. Just like data is measured in terms of discrete data packets, energy is also quantified or measured in discrete units of "energy packets" [25], [26], [37], so that an energy packet is defined as the minimum amount of energy needed to transmit a single data packet. Ideally the energy flow into the device, expressed as discrete units of energy per unit time, should somehow balance the flow of data packets that are being generated and then assembled (for instance based on sensing) so that packet transmission is not held up. Thus energy harvesting should ideally match the needs of the data transmission workload. Within this framework:

- We first show the counterintuitive result that if the flows of energy and of data packets are exactly balanced, the system exhibits an unstable behaviour such that both the backlog of packets and the amount of stored energy saturate their respective buffers while the effective work carried out by the transmitter tends to zero. We also derive results concerning the performance of the system when the energy and work flows are unbalanced. In reality, all buffers are of finite capacity, so we thoroughly study the finite capacity case. In the process, we address a fundamental problem in computer science regarding the stability of the *join* synchronisation primitive [38], [39], [40] when the synchronisation time is negligibly small.
- In a second part we use the G-Network paradigm to formulate the flow of renewable energy among multiple energy storage sub-systems and multiple consumers. In this part we focus on the optimum flow or dispatching of energy so as to maximise or minimise different utility functions of interest. In particular, we derive gradient based polynomial time and memory complexity algorithms to this effect, and illustrate the work with several numerical examples.

II. System Model

Consider a system that at some time t contains $K(t)$ packets to transmit and $M(t)$ energy packets in its storage. We use the term "energy packet" [25], [26] along the terminology developed in some recent work where energy is assumed to be composed of discrete units. We assume that an energy packet is exactly the amount of energy needed to transmit a data packet. Assume also that the system has a data buffer of size B data packets, while its energy storage device can store up to E energy packets. The state of the system is then described by the pair $(K(t), M(t))$ and we shall be interested in its probability distribution $p(n,m,t) = Pr[K(t) = n, M(t) = m)]$, which will be analysed using standard methods that are usually associated with queueing networks [42], [43].

The formation of packets and their placement in the packet buffer is assumed to follow a Poisson process of rate λ while the energy packets are also assumed to arrive at random into the energy storage device (e.g., a capacitor or rechargeable battery) as a Poisson process of rate Λ. Note that Λ would correspond to some rate of a joules per second, where a is the number of joules required to transmit one packet.

In such electronic systems, the rate at which energy is harvested is slow compared with the rate at which packets can be transmitted by the electronic device. For instance, it may take many milliseconds to get enough energy from the harvester while it may just take a few microseconds, or more likely a few nanoseconds, to transmit a packet. Similarly, in such a device the rate at which packets are formed must not exceed the rate at which energy is being harvested. Note that an energy packet is exactly the amount of energy needed to transmit a packet. We can assume that if the energy needed is available, the time needed for the transmission of a packet is negligible compared with the inter-arrival times of energy packets and data packets to the device. As a consequence, if the system perchance enters some state $(K(t), M(t))$ with $K(t) > 0,\ M(t) > 0$, then it will instantaneously (in zero time) transit to the either the state $(0, M(t) - K(t))$ if $M(t) \geq K(t)$, or the state $(K(t) - M(t), 0)$ if $K(t) \geq M(t)$.

Thus we only need to consider for $p(n,m,t)$ the state space S of pairs of integers (n,m):

$$S = \{\ (0,0),\ (n,0),\ (0,m)\ :n > 0,\ m > 0\ \} \tag{1}$$

III. The System without Energy Storage

Consider a system that stores packets in buffer of infinite capacity but has no means to store energy such that if energy is harvested but not immediately consumed, it is immediately lost, *i.e.*, $E = 0$. In this case, the data buffer behaves as a queue with Poisson arrivals, and exponentially distributed service times of rate Λ [**?**]. Note that in this case, the arrival corresponds to the data packets, while the "services" are in fact the arrivals of energy packets. We have also assumed that the transmission time of a data packet, typically in nanoseconds, is extremely short compared with the other times involved (packet inter-arrival times for both data and energy packets), since both the data gathering or sensing process and the energy harvesting process will take much longer. For instance, if sensing is related to temperature, we will be dealing of successive minutes before a change of temperature is sensed, and if we are sensing human movement we would need tens of milliseconds to note any changes due to the speed at which people move. Energy harvesting will also be relatively slow simply because of the very minute amount of energy that can be collected.

Thus, if $\Lambda \geq \lambda$, the throughput of data packets is simply the arrival rate of packets. Furthermore, when $\Lambda > \lambda$, then the data packet buffer will have a queue length $b(t)$ whose probability distribution $p_d(n,t) = Pr[b(t) = n]$ is given in steady state by:

$$p_d(n) \equiv \lim_{t \to \infty} p_d(n,t) = (\frac{\lambda}{\Lambda})^n (1 - \frac{\lambda}{\Lambda}) \tag{2}$$

The energy wastage by this system in steady state is then:

$$L_e = \Lambda p_d(0) = \Lambda - \lambda \tag{3}$$

as intuitively expected.

IV. The System without Data Storage

Similarly we can consider the opposite case, where the system is able to store energy in a battery of infinite capacity but unable to store data packets, so that if a data packet arrives when there is no available energy then that data packet is simply lost, *i.e.*, $B = 0$.

Then the probability distribution $p_e(m,t) \equiv Pr[e(t) = m]$ for the variable $e(t)$ representing the number of energy packets that are currently stored in the system, and in steady state it will be given by:

$$p_e(m) \equiv \lim_{t \to \infty} p_e(m,t) = (\frac{\Lambda}{\lambda})^m (1 - \frac{\Lambda}{\lambda}) \tag{4}$$

if $\Lambda < \lambda$, when the energy production rate is less than the data packet arrival rate. The equation below gives the steady state loss rate of data packets that arrive to the system, cannot be stored and are therefore discarded rather than transmitted because there is no energy available:

$$L_d = \lambda p_e(0) = \lambda - \Lambda \tag{5}$$

V. The System with Finite Capacity Energy and Data Buffers

When both the data buffer and the energy storage capacity are non-zero, the analysis does not fit into a known standard model but it is straightforward to carry out. Let us just write the balance equations for $p(n,m,t)$ in steady state so that we may drop the dependency on t and just write $p(n,m) = \lim_{t \to \infty} Pr[K(t) = n, M(t) = m]$, assuming that the equilibrium equation actually exists, which should be, since we are dealing with finite buffer and energy capacity and hence the model is a finite Markov chain.

The balance equations are as follows. The first one deals with what happens around the state $(0,0)$, which can be reached if either there was just one data packet and it was transmitted as soon as an energy packet arrived, or there was one energy packet and it was consumed as soon as a data packet arrived:

$$p(0,0)[\lambda + \Lambda] = \Lambda p(1,0) + \lambda p(0,1) \tag{6}$$

For $0 < n < B$ we have:

$$p(n,0)[\lambda + \Lambda] = \Lambda p(n+1,0) + \lambda p(n-1,0) \tag{7}$$

while:

$$p(B,0)\Lambda = p(B-1,0)\lambda \tag{8}$$

We note that these equations have a solution of the form:

$$p(n,0) = \alpha^n C_d, \ \alpha = \frac{\lambda}{\Lambda} \tag{9}$$

where C_d is a constant.

Similarly for the energy storage system we will have for $0 < m < E$:

$$p(0,m)[\lambda + \Lambda] = \Lambda p(0,m-1) + \lambda p(0,m+1) \tag{10}$$

while:

$$p(0,E)\lambda = p(0,E-1)\Lambda \tag{11}$$

We note that these equations have a solution of the form:

$$p(0,m) = \beta^m C_e, \ \beta = \frac{\Lambda}{\lambda} \tag{12}$$

for some constant C_e.

Since the Markov chain with state probabilities $\{p(n,m,t) : (n,m) \in S;\ t \geq 0\}$ is irreducible and aperiodic, the stationary probability distribution exists and is unique, hence $C_e = C_d = p(0,0)$ and:

$$1 = p(0,0)[1 + \sum_{n=1}^{B} (\frac{\lambda}{\Lambda})^n + \sum_{m=1}^{E} (\frac{\Lambda}{\lambda})^m] \quad (13)$$

$$= p(0,0)[-1 + \frac{1-\alpha^{B+1}}{1-\alpha} + \frac{1-\beta^{E+1}}{1-\beta}] \quad (14)$$

so that

$$p(0,0) = \frac{2-\alpha-\beta}{\alpha^B(1-\alpha) + \beta^E(1-\beta)} \quad (15)$$

and since $\beta = (\alpha)^{-1}$ we have:

$$p(0,0) = \frac{(1-\alpha)\alpha^E}{1-\alpha^{B+E+1}}. \quad (16)$$

Hence:

$$p(n,0) = \alpha^n \frac{2-\alpha-\beta}{\alpha^B(1-\alpha) + \beta^E(1-\beta)},\ 0 \leq n \leq B \quad (17)$$

$$p(0,m) = \beta^m \frac{2-\alpha-\beta}{\alpha^B(1-\alpha) + \beta^E(1-\beta)},\ 0 \leq m \leq E \quad . \quad (18)$$

A. *Marginal Probabilities*

From the previous analysis, we also know that:

$$p(n,0) = \alpha^n p(0,0) \quad 0 < n \leq B \quad (19)$$

$$p(0,m) = \beta^m p(0,0) \quad 0 < m \leq E \quad (20)$$

so that $p(n,0) > 0$ and $p(0,m) > 0$ only if $p(0,0) > 0$.

Thus the marginal probabilities for the queue length of data packets is

$$p_d(n) = \sum_{m=0}^{\infty} p(n,m) = p(n,0),\ n > 0 \quad (21)$$

$$p_d(0) = \sum_{m=0}^{\infty} p(n,m) = \frac{1-\beta^{E+1}}{1-\beta} p(0,0) \quad (22)$$

and similarly $p_e(m) = p(0,m),\ m > 0$, while

$$p_e(0) = \frac{1-\alpha^{B+1}}{1-\alpha} p(0,0) \quad (23)$$

and:

$$p_d(n) = \alpha^n p(0,0),\ 0 < n \leq B \quad (24)$$

$$p_e(m) = \beta^m p(0,0),\ 0 < m \leq E \quad (25)$$

or

$$p_d(n) = \frac{\alpha^{E+n}(1-\alpha)}{1-\alpha^{E+B+1}},\ 0 < n \leq B \quad (26)$$

$$p_e(m) = \frac{\beta^{B+m}(1-\beta)}{1-\beta^{E+B+1}},\ 0 < m \leq E \quad (27)$$

VI. Energy Wastage and Data Packet Loss with Finite Storage Capacities

When the energy storage capacity is finite, or the data packet buffer is finite, as with the case we studied earlier where the buffers were of zero size, we are bound to have some level of energy loss or of data packet loss. The loss rates $L_e,\ L_d$ in energy and data packets per second, respectively, can be easily computed as:

$$
\begin{aligned}
L_e &= \Lambda \sum_{n=0}^{\infty} p(n,E) = \Lambda p(0,E) \qquad (28)\\
&= \Lambda \beta^E \frac{2-\alpha-\beta}{\alpha^B(1-\alpha)+\beta^E(1-\beta)}\\
L_d &= \lambda \sum_{m=0}^{\infty} p(B,m) = \lambda p(B,0) \qquad (29)\\
&= \lambda \alpha^B \frac{2-\alpha-\beta}{\alpha^B(1-\alpha)+\beta^E(1-\beta)}
\end{aligned}
$$

so that after some algebra we get:

$$
\begin{aligned}
L_e &= \frac{\Lambda-\lambda}{1-\alpha^{B+E+1}} \qquad (30)\\
L_d &= \frac{\lambda-\Lambda}{1-\beta^{B+E+1}} \qquad (31)
\end{aligned}
$$

A. When Buffer Sizes Are Very Large

Now if $\alpha < 1$, so that energy is more plentiful than needed, we see that if B or E or both tend to infinity, then $L_e \rightarrow \Lambda - \lambda$, while $L_d \rightarrow 0$, as would be expected.

Similarly when $\beta < 1$ or $\alpha > 1$, then when B or E or both tend to infinity, we have $L_d \rightarrow \lambda - \Lambda$, while $L_d \rightarrow 0$.

B. When Energy Supply Exactly Matches the Data Flow

An interesting case arises when the energy packets balance the data packets exactly, *i.e.*, $\alpha = \beta = 1$ or $\lambda = \Lambda$. In this case we have:

$$
L_e = L_d = \frac{\lambda}{B+E+1} \qquad (32)
$$

which obviously tends to zero when B or E or both tend to infinity.

C. The Case with Infinite Data Buffer and Finite Energy Buffer

Now assume that $E < \infty$ (finite energy buffer) and let the data buffer be unlimited, *i.e.*, $B \rightarrow \infty$. In this case we see from Equation (6) that as long as $\alpha < 1$, and of course $\beta > 1$ so that the supply of harvested energy *exceeds* the needs of the flow of data packets, then:

$$
\begin{aligned}
p(n,0) &= \alpha^n(1-\alpha)\alpha^E,\ n \geq 0 \qquad (33)\\
p(0,m) &= \beta^m(1-\alpha)\alpha^E,\ E \geq m \geq 0 \qquad (34)
\end{aligned}
$$

so that the average number of data packets waiting to be transmitted is:

$$
\Pi_D = \alpha^E \frac{\alpha}{1-\alpha} \qquad (35)
$$

We see as expected that Pi_D decreases as α becomes smaller, and also as E becomes larger.

D. When the Data Buffer is Finite and the Energy Buffer is Infinite

Now assume that $B < \infty$ (finite data buffer) and let the energy buffer be unlimited, *i.e.*, $E \rightarrow \infty$. We see from Equation (6) that if $\beta < 1$, and of course $\alpha > 1$ so that the supply of harvested energy *is less than* the needs of the flow of data packets, then:

$$
\begin{aligned}
p(n,0) &= \alpha^n(1-\beta)\beta^B,\ B \geq n \geq 0 \qquad (36)\\
p(0,m) &= \beta^m(1-\beta)\beta^B,\ E \geq m \geq 0 \qquad (37)
\end{aligned}
$$

so that the average number of energy packets that are waiting to be used is:

$$
\Pi_E = \beta^B \frac{\beta}{1-\beta} \qquad (38)
$$

and we see that Π_E decreases as β decreases, and also decreases when B increases for $\beta < 1$.

VII. System Stability and Instability

The stability question for the considered system is of interest when the data and energy storage buffers are unlimited. In this case it is of interest to determine whether the backlog of data packets, or the backlog of stored energy, or both, remain finite or not and under what circumstances. Obviously the question does not arise if we consider a finite time horizon, since starting from an empty system at $t = 0$, *i.e.*, with $p(0, 0, 0) = 1$, the probability that the backlog is infinite for any finite t is simply zero because of the Poisson arrivals assumption for both the data and the energy packets.

Definition 1. *We will say that the* system is stable *if as $t \to \infty$, and for $B \to \infty$ and $E \to \infty$, the probability that the backlog of data and energy packets remains finite with probability one. Otherwise the system will be said to be unstable.*

Definition 2. *The* system is stable with respect to data packets *(or energy packets) if the number of data packets (or energy packets) is finite with probability one as $t \to \infty$, $B \to \infty$ and $E \to \infty$. Otherwise we say that the system is unstable with respect to the data packets (or energy packets).*

Now for a finite G and H with $0 \leq G < B$ and $0 \leq H < E$, let us define the probabilities that the respective backlogs of data and energy packets do not exceed G and H, respectively, in steady state:

$$P_d(G) = lim_{t\to\infty} Prob[0 \leq N(t) \leq G \leq B] \tag{39}$$

$$P_e(H) = lim_{t\to\infty} Prob[0 \leq M(t) \leq H \leq E] \tag{40}$$

so that using our previous results from Section V-A, we have:

$$P_d(G) = p_d(0) + \sum_{n=1}^{G} p_d(n) = \frac{\alpha^G(1-\alpha) + \beta^E(1-\beta)}{\alpha^B(1-\alpha) + \beta^E(1-\beta)} \tag{41}$$

$$P_e(H) = p_e(0) + \sum_{m=1}^{H} p_e(m) = \frac{\beta^H(1-\beta) + \alpha^B(1-\alpha)}{\alpha^B(1-\alpha) + \beta^E(1-\beta)} \tag{42}$$

leading directly to the following results.

Theorem 1. *If $\beta > 1$, and hence $\alpha < 1$, we see from Equation (41) that as $E \to \infty$ and $B \to \infty$, $P_d(G) \to 1$ for all finite G, and the system is stable with respect to the data packets. However for $\beta > 1$ and $\alpha < 1$, we see from Equation (42) that $P_e(H) \to 0$ for all finite H, and the system is unstable with respect to the energy packets.*

Theorem 2. *If $\beta < 1$, and hence $\alpha > 1$, we see from Equation (42) that as $E \to \infty$ and $B \to \infty$, $P_e(H) \to 1$ for all finite H, and the system is stable with respect to the energy packets. However under the same conditions, Equation (41) indicates that $P_d(G) \to 0$ for all finite G, hence the system is unstable with respect to the data packets.*

These two results now tell us the following:

Corollary 1. *If $\lambda \neq \Lambda$ so that either $\alpha > 1$ or $\beta > 1$, it follows that the system is unstable and when the data and energy buffers are unlimited, in steady state either the number of data packets or the number of energy packets is unbounded with probability one.*

VIII. Stability and Instability When $\lambda = \Lambda$

Intuitively, one can imagine that the best operating conditions are when both of the resources, energy and data packets, are flowing into the system at equal rates so that the energy is exactly the amount needed for packet transmission, and the packets consume all the energy in the system. In fact, a more careful analysis tells us that things are somewhat complicated.

On the one hand, we see that when $\lambda = \Lambda$ or $\alpha\beta = 1$, the expression for $p(0, 0)$ is an indeterminate form, so that we apply L'Hôpital's rule and take the derivative of the numerator and denominator to obtain:

$$\lim_{\alpha\to 1} p(0,0) = \lim_{\alpha\to 1} \frac{E\alpha^{E-1} - (E+1)\alpha^E}{-(B+E+1)\alpha^{B+E}} \tag{43}$$

$$= \frac{1}{B+E+1} \tag{44}$$

As a consequence we get the following result:

Theorem 3. *If $\alpha = 1$ and either $E \to \infty$ or $B \to \infty$, or both, then $p(0, 0) \to 0$, so that both $p(n, 0) \to 0$ and $p(0, m) \to 0$ for all finite (n, m).*

Also, Equations (41) and (42) at $\alpha = 1$ are indeterminate forms, such that applying L'Hôpital's rule we obtain:

$$\lim_{\alpha \to 1} p_d(n) = \frac{1}{B+E+1}, \ 0 < n \leq B \tag{45}$$

$$\lim_{\alpha \to 1} p_e(m) = \frac{1}{B+E+1}, \ 0 < m \leq E \tag{46}$$

Hence, we can compute the average values of the data $< D >$ and energy $< E >$ backlog for $\alpha = 1$ in a simple manner.

Theorem 4. *If* $\alpha = 1$ *then:*

$$< D >= \frac{B(B+1)}{2(B+E+1)}, \ < E >= \frac{E(E+1)}{2(B+E+1)} \tag{47}$$

Thus, as $B \to \infty$ *the average data backlog tends to infinity, and when* $E \to \infty$ *the average backlog of energy tends to infinity.*

We can also write:

$$lim_{\alpha \to 1} P_d(G) = \frac{G+E+1}{B+E+1}$$
$$lim_{\alpha \to 1} P_e(H) = \frac{H+B+1}{B+E+1}$$

which tells us that:

Theorem 5. *For any finite* G *and* H *when* $\Lambda = \lambda$*, as* B *and* E *tend to infinity, the probability that the data and energy backlogs remain finite tends to zero.*

IX. A Single Harvesting Node with Data Packet Retransmission

Clearly once an energy packet has been used to transmit a data packet, it cannot be forwarded to some other node since it has been used up. On the other hand, we may need to retransmit the same data packet if an error is detected during the transmission, or if we wish to reduce the probability of packet errors by sending several duplicates. Such situations lead to interesting modelling issues that we will dwell on in this section. Let λ and Λ be the arrival rates of data and energy packets, respectively. Consider a single harvesting and transmission node, and suppose that:

- If a data packet is waiting in queue and an energy packet arrives such that the data packet can be transmitted, then with probability π one energy packet will not suffice and the data packet remains in the node. When another energy packet arrives at some later time, the process will repeat itself independently of the previous event.
- Also, if a data packet arrives to the node when one or more energy packets are waiting, then it is immediately transmitted but the transmission may be unsuccessful and another energy packet will be needed with probability p. This process may be repeated independently of the previous outcome with the same probabilities of success $(1-p)$ and failure p.

Now we see that the state space is still $S = \{(0,0),(n,0),(0,m): \ n,m \geq 1 \ \}$ but the possible state transitions differ from those of the previous section. We still have the following state transition rates:

- - $\lambda: \ for \ (n,0) \to (n+1,0), \ n \geq 0$, when a data packet arrives, and
- - $\Lambda: \ for \ (0,m) \to (0,m+1), \ m \geq 0$, when an energy packet arrives.

Also, when an energy packet arrives to an empty energy buffer, we will have:

- - $\Lambda\pi: \ for \ (n,0) \to (n,0), \ n \geq 1$, because upon arrival of an energy packet to an empty energy buffer, if the data packet transmission is immediately followed by the request for another data packet transmission with probability π, then the new data packet will not be transmitted due to the lack of an energy packet and will simply replace the previous one. However, if the data packet transmission (caused by the arrival of an energy packet) is not followed by another data packet transmission request, which occurs with probability $(1-\pi)$, then we have the transitions:
- - $\Lambda(1-\pi): \ for \ (n,0) \to (n-1,0), \ n \geq 1$, and we will also have the transition:
- - $\lambda p: \ for \ (0,1) \to (1,0)$, when a data packet arrives, because the energy packet in queue serves the transmission of the arriving energy data packet, an additional data packet transmission is needed with probability p, and the data packet will have to wait for another energy packet to arrive.

Finally, there will be a set of transitions starting from states of the form $(0,m), \ m > 0$, when the arrival of a data packet results in a sequence of k successive repetitions of data packets arriving to the node with probability $p^k(1-p)$ where $m \geq k \geq 0$, namely:

- - $\lambda(1-p): \ for \ (0,m) \to (0,m-1), \ m > 0$,
- - $\lambda p^{k-1}(1-p): \ for \ (0,m) \to (0,m-k), \ m \geq k > 1$, and finally in the last case below, the arriving data packet reduces the number of energy packets by 1 and generates one additional data packet that is also transmitted, then there may be another data packet transmission request and so on, but when all the energy packets are depleted, the $m-th$ and final transmission request cannot be satisfied and the system moves into state $(1,0)$ having depleted all its energy packets and having one final data packet waiting to be transmitted:

- - $\lambda p^m: \; for \; (0,m) \rightarrow (1,0), \; i.e., \; k=m$, and notice that for any $m>0$ the sum of these probabilities is one:

$$\sum_{k=0}^{m-1} p^k(1-p) + p^m = 1 \tag{48}$$

The above transitions lead to the equilibrium equations:

$$\begin{aligned}
p(0,0)[\lambda+\Lambda] &= \lambda \sum_{l=1}^{\infty} p^{l-1}(1-p)p(0,l) + \Lambda(1-\pi)p(1,0) \\
p(1,0)[\lambda+\Lambda(1-\pi)] &= \lambda \sum_{l=0}^{\infty} p^{l}p(0,l) + \Lambda(1-\pi)p(2,0) \\
p(n,0)[\lambda+\Lambda(1-\pi)] &= \lambda p(n-1,0) + \Lambda(1-\pi)p(n+1,0), \; n>1 \\
p(0,m)[\lambda+\Lambda] &= \lambda \sum_{l=1}^{\infty} p^{l-1}(1-p)p(0,m+l) + \Lambda p(0,m-1), \; m>0
\end{aligned} \tag{49}$$

Regarding the stability of this system, we cannot use results such as the conditions of *Foster–Pakes–Tweedie* [44] because we are dealing with two (or more in the network case) "queues", namely the energy packet backlog and the data packet backlog. However, we are able to prove the following useful result.

Theorem 6. *If $p > \pi \geq 0$ and $\Lambda(1-p) < \lambda < \Lambda(1-\pi)$, the stationary distribution exists and is of the form:*

$$\begin{aligned}
p(0,m) &= p(0,0)Q^m, m \geq 1 \\
with \; Q &= \frac{\Lambda}{\lambda+p\Lambda} \\
q &= \frac{\lambda}{\Lambda(1-\pi)} \\
p(n,0) &= p(1,0)q^{n-1}, \; n \geq 1 \\
p(1,0) &= \frac{\lambda+p\Lambda}{\Lambda(1-\pi)}p(0,0) \\
p(0,0) &= \frac{[\Lambda(1-\pi)-\lambda][\lambda-\Lambda(1-p)]}{\Lambda(p-\pi)[\lambda+p\Lambda]}
\end{aligned} \tag{50}$$

Proof: To proceed with the proof, we substitute Equation (50) in Equation (49), which after some algebra becomes:

$$\begin{aligned}
p(0,0)[\lambda+\Lambda] &= \frac{\lambda Q(1-p)}{1-pQ}p(0,0) + \Lambda(1-\pi)p(1,0) \\
p(1,0)[\lambda+\Lambda(1-\pi)] &= \frac{\lambda}{1-pQ}p(0,0) + \Lambda(1-\pi)qp(1,0) \\
[\lambda+\Lambda(1-\pi)] &= \frac{\lambda}{q} + \Lambda(1-\pi)q \\
[\lambda+\Lambda] &= \frac{\lambda Q(1-p)}{1-pQ} + \frac{\Lambda}{Q}
\end{aligned} \tag{51}$$

After completing the substitutions of the values of Q and p we have:

$$\begin{aligned}
p(0,0)[\lambda+\Lambda] &= \Lambda(1-p)p(0,0) + \Lambda(1-\pi)p(1,0) \\
p(1,0)[\lambda+\Lambda(1-\pi)] &= [\lambda+p\Lambda]p(0,0) + \lambda p(1,0) \\
[\lambda+\Lambda(1-\pi)] &= \Lambda(1-\pi) + \lambda \\
[\lambda+\Lambda] &= \Lambda(1-p) + \lambda + p\Lambda
\end{aligned} \tag{52}$$

It is clear that the last two equations are satisfied, while the first two equations both reduce to:

$$p(1,0) = \frac{\lambda+p\Lambda}{\Lambda(1-\pi)}p(0,0)$$

Now using the fact that the probabilities must sum to one we have:

$$p(0,0) = \frac{[\Lambda(1-\pi)-\lambda][\lambda-\Lambda(1-p)]}{\Lambda(p-\pi)[\lambda+p\Lambda]}$$

which completes the proof. ∎

Remark 1. *Based on the above results, we notice that:*

- *The probability that the data queue is empty can be obtained as:*

$$P[n=0] = p(0,0) + \sum_{m=1}^{\infty} p(0,m) = \frac{\Lambda(1-\pi)-\lambda}{\Lambda(p-\pi)} \quad (53)$$

- *The probability that the data queue is non-empty is:*

$$P[n>0] = \sum_{n=1}^{\infty} p(n,0) = \frac{\lambda-\Lambda(1-p)}{\Lambda(p-\pi)} \quad (54)$$

The probability that the data queue is either full or empty is then obviously:

$$P[n \geq 0] = 1 = \frac{\Lambda(1-\pi)-\lambda+\lambda-\Lambda(1-p)}{\Lambda(p-\pi)} \quad (55)$$

X. Energy Packet Networks

The EPN model for networks of energy prosumers combines:

- The intermittent flow of harvested of energy, represented by a random flow of arriving "energy packets (EPs)". An energy packet is a discrete unit of energy, e.g. X Joules, which is represented as arriving in "one chunk". It is simply a simplified version of the actual flow of energy which is continuous.
- The storage of energy (say in batteries), again in discrete units of EPs. An energy storage unit (ES) such as a battery is modelled as a queue of EPs that are waiting to be used. The ES is replenished by a flow of EPs from some external source including an energy harvesting unit, and it can be depleted both when energy is forwarded to a consumer, and through losses that represent leakage or line losses.
- The sources of power, the ESs and the consumers are interconnected by Power Switches (PSs) which dynamically connects the sources of power to the ESs, and the sources of power and ESs to the consumers.
- The consumers will request for EPs from either a PS or ESs, and these requests will typically be intermittent, since they are a function of the work that these consumers accomplish with the energy.

A typical example of such consumers are ICT systems which intermittently receive computational work to accomplish (for instance in terms of programs that need to be executed, or in terms of data packets that need to be transmitted), and which in turn require energy to accomplish this work. Thus a consumer is also represented by:

- A queue of work that it has to accomplish which may originate from some outside source or from some other consumer of energy which accomplished a prior work step, and
- One or more servers that have job execution times which have a random duration which may depend on the nature of the job and the speed of the work unit, and also on the flow of power to the server.

An EPN will contain multiple work units with external arrivals of jobs or data packets, the jobs or data packets may move from one consumer queue to the other, and there will be multiple queues of EPs, with external arrivals of renewable or steadily flowing energy, and energy itself may flow between ES, or it may leak from the energy stores, or move to consumers where it is used to execute jobs or transmit packets. Such EPNs can be analysed using a branch of queueing network theory called G-Networks [45], [46]. Typically in queueing theory, jobs move around a set of servers where they queue up to receive service until they complete their needs and leave the system. In G-Networks on the other hand, some of the flows of "customers", in this case the EPs, can queue up (as in an ES), but they can move to act as servers or activators for other customers, in this case the jobs that need to be executed or the data packets that need to be transmitted, that queue up at the servers where work is accomplished. Thus the G-Network can be used to represent job flows that consume energy and energy flows that allow jobs to be executed.

Electronic systems that capture a similar physical behaviour have been recently built and experimentally tested under the name of "power packet" systems. They are designed for the smart dispatching of electrical energy [47], [48], and in some approaches the forwarding of an EP takes the form of a pulse of current with a fixed voltage and duration, and each EP is equipped with an encoded header compromising the destination information for the EP [49] that allows PSs to determine where the energy needs to be sent.

In previous work [50] we have used the EPN model to study the architectures that interconnect energy prosumer systems so that energy losses and the response time to service request are minimised.

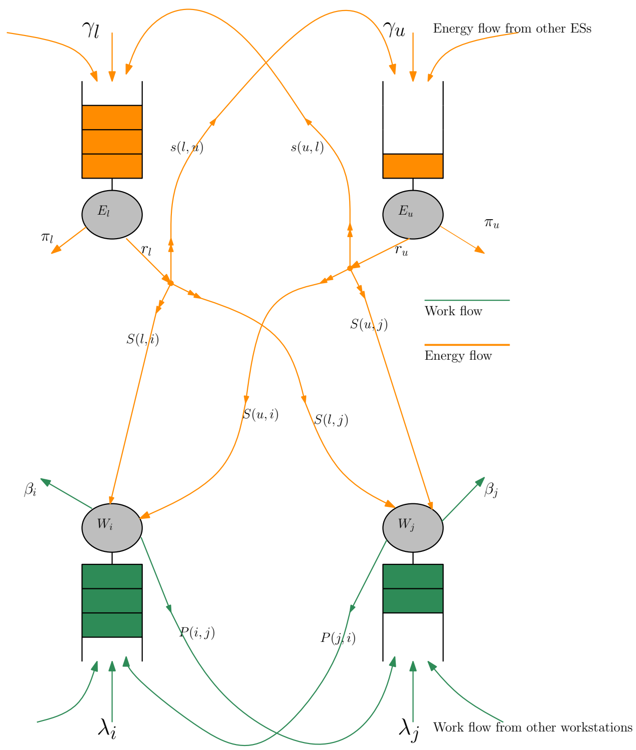

Fig. 1. Schematic representation of an EPN with two types of nodes and "queues": the "positive" nodes contain the work to be done while the "negative" nodes are the ESs which store energy and are replenished by harvesting. ESs provide power in the form of EPs to the "positive" nodes. Work can move from one positive node to another where they are processed when energy becomes available, and finally the work leaves the network after completing a certain number of work steps. EPs can leak from the ESs (negative nodes), or be transferred to another ES or they can be transferred to a positive node to accomplish work.

In the following we develop optimisation algorithms that optimise utility functions that include a linear combination of the throughput or useful, and the the probability that the system does *not* run out of energy. Thus the type of utility that we seek to *maximise* includes two sets of terms: the sum of the job processing rates of the system, and the sum of the probabilities that there is reserve energy in the system. Clearly if we try to just maximise the work done we may run out of energy, while if we try to just save energy then the system may delay the job execution times.

XI. The Mathematical Model

We study an EPN consisting of a finite set of nodes. It contains m energy storage (ES) nodes denoted E_l, $1 \leq l \leq m$ store and dispatch EPs, while n workstation (WS) nodes W_i, $1 \leq i \leq n$ are "places of work" where jobs are executed. The system diagram is described schematically in Figure 1. Energy packets enter the energy store E_l at rate γ_l from external renewable sources. Also, the energy stored in E_l may be lost at a rate π_l due to leakage and standby losses, and E_i sends energy packets out of the store at rate r_l either to other energy stores or to workstations.

The external arrival rate of jobs to the work station W_i is λ_i, while the workstation may also receive jobs from another workstation W_j with probability P_{ji} after workstation W_j finishes processing that job. Moreover, uncompleted jobs stored at the work station W_i might also be lost at a time-out rate β_i.

However processing at W_i can only occur if an energy packet arrives at the workstation from some ES E_l. This can happen in two ways:

- With probability $s(l,i)$, E_l sends an energy packet to W_i, and it may send EPs to E_k with probability $S(l,k)$, with $\sum_{l=1}^{n} s(l,i) + \sum_{k=1}^{m} S(l,k) = 1$ for any l. We will write $w(l,i) = r_l s(l,i)$ and $W(l,k) = r_l S(l,k)$ so that $r_l = \sum_{l=1}^{n} w(l,i) + \sum_{k=1}^{m} W(l,k)$.
- With probability P_{ij} the workstation W_i sends the job that has just completed to node W_j for more processing, or with probability $d_i = 1 - \sum_{j=1}^{n} P_{ij}$ the workstation W_i will simply remove the job that terminates at W_i from the system without forwarding it to another workstation.

In fact a more complex model in which a single job can consume more than one energy packet at a time can also be used, and will be considered in future work.

As a consequence of the above assumptions, using G-Network theory [45], [51] the probabilities Q_i that W_i has at least one job in its queue, and q_l that E_l has at least one EP in its ES, will satisfy:

$$Q_i = \frac{\lambda_i + \sum_{j=1}^{n}\sum_{l=1}^{m} q_l w(l,j) Q_j P_{ji}}{\sum_{l=1}^{m} q_l w(l,i) + \beta_i}, \tag{56}$$

where the rate of service at W_i is determined by the rate at which it receives energy, and q_l is the probability that energy storage station E_l has at least one EP in store:

$$q_l = \frac{\gamma_l + \sum_{k=1}^{m} q_k W(k,l)}{r_l + \pi_l}. \tag{57}$$

The above equations also allow us to state the following result [51]:

Theorem 1 *Assume that both external arrivals of jobs at rates λ_i, and of EPs from renewable energy sources at rates γ_l, are independent Poisson processes. Suppose that the r_l, π_l and β_i are the parameters of independent exponentially distributed random variables. Then if $(K_1, \ldots, K_n) \geq (0, \ldots, 0)$ and $(k_1, \ldots, k_m) \geq (0, \ldots, 0)$ represent the backlogs of jobs to be executed at the workstations, and the number of energy packets stored at the ESs, respectively. Then*

$$\begin{aligned} &\lim_{t\to\infty} P[K_1(t) = K_1, \ldots, K_n(t) = K_n; \\ &\qquad k_1(t) = k_1, \ldots, k_m(t) = k_m] \\ &= \prod_{i=1}^{n} Q_i^{K_i}(1-Q_i) \prod_{i=1}^{m} q_l^{k_l}(1-q_l). \end{aligned}$$

Corollary 1 *Because in steady-state the joint probability distribution of the number of jobs waiting, and energy packets stored, at each of the WS and ES, is the product of the marginal distributions, we have:*

$$\lim_{t\to\infty} P[K_i(t) \geq K_i] = Q_i^{K_i}, \ \lim_{t\to\infty} P[k_l(t) \geq k_l] = q_l^{k_l}.$$

A. Vector Representation

Denote by Q and q, respectively, the row vectors whose elements are (56) and (57). We can then write:

$$Q = \Lambda + Q w_q P; \ q = \Gamma + qW, \tag{58}$$

where

- w_q is the $n \times n$ diagonal matrix whose diagonal elements are $\sum_{l=1}^{m} q_l w(l,j)$,
- P is the $n \times n$ matrix
$$P = [\frac{P_{ij}}{\sum_{l=1}^{m} q_l w(l,j) + \beta_j}]_{n\times n},$$
- W is the $m \times m$ matrix
$$W = [\frac{W(k,l)}{r_l + \pi_l}]_{m\times m},$$
- and the $\Lambda, \ \Gamma$ are, respectively, the n and m dimensional vectors of elements
$$\Lambda_i = \frac{\lambda_i}{\sum_{l=1}^{m} q_l w(l,i) + \beta_i}, \ \Gamma_l = \frac{\gamma_l}{r_l + \pi_l},$$

The expression (58) can then be readily written as:

$$q = \Gamma[1-W]^{-1}, \quad Q = \Lambda[1 - w_q P]^{-1}. \tag{59}$$

XII. An Interesting Special Case

A case of practical interest arises when we take a specific “high level” optimisation decision, namely:

- All energy stores should, on average, contain the same amount of energy, so that if any of them fails the others can best support the system, so that:
$$q_* \equiv \frac{\gamma_l + \sum_{k=1}^{m} q_* W(k,l)}{r_l + \pi_l}, \ \forall l,$$
- The backlog of work at each of the workstations should on average be the same, i.e.
$$Q_* \equiv \frac{\lambda_i + \sum_{j=1}^{n}\sum_{l=1}^{m} q_* w(l,j) Q_* P_{ji}}{\sum_{l=1}^{m} q_* w(l,i) + \beta_i}, \ \forall i.$$

Let us denote $W^+(l) = \sum_{k=1}^{m} W(l,k)$, and $w^+(i) = \sum_{l=1}^{m} w(l,i)$. Then we have:

$$\begin{aligned} q_* &= \frac{\gamma_l}{r_l + \pi_l - W^+(l)}, && (60)\\ Q_* &= \frac{\lambda_i}{\beta_i + q_*[w^+(i) - \sum_{j=1}^{n} w^+(j)P_{ji}]}, && (61)\\ &= \frac{\lambda_i}{\beta_i + \frac{\gamma_l}{r_l+\pi_l-W^+(l)}[w^+(i) - \sum_{j=1}^{n} w^+(j)P_{ji}]}, \end{aligned}$$

and we know that $w^+(i) - \sum_{j=1}^{n} w^+(j)P_{ji} \geq 0$, and we recall that $Q_*,\ q_*$ must satisfy $0 < Q_* < 1$ and $0 < q_* < 1$. If q_* and Q_* are fixed, we obtain:

$$\begin{aligned} w^+(i) - \sum_{j=1}^{n} w^+(j)P_{ji} = \frac{1}{q_*}[\frac{\lambda_i\beta_1}{\lambda_1} - \beta_i] \\ + \frac{\lambda_i}{q_*\lambda_1}[\frac{\lambda_1}{Q_*} - \beta_1] = \frac{1}{q_*}[\frac{\lambda_i}{Q_*} - \beta_i], \end{aligned} \quad (62)$$

so that we must have:

$$Q_* < \frac{\lambda_i}{\beta_i},\ \forall\ i,\ or\ Q_* < \min_{1\leq i\leq n}[\frac{\lambda_i}{\beta_i}]. \quad (63)$$

Then writing the m-vector c with elements:

$$c_i = \frac{1}{q_*}[\frac{\lambda_i}{Q_*} - \beta_i], \quad (64)$$

we have:

$$\begin{aligned} w^+ &= w^+P + c, && (65)\\ &= c[1-P]^{-1}, \end{aligned}$$

which has a unique solution because the Markov chain P is transient. Since all the $q_k = q_*$ are identical, we set

$$w(k,i) = \frac{w^+(i)}{m}, \quad (66)$$

so that all the $w(k,i)$ are now determined.

As a consequence, for each $1 \leq k \leq m$ we have:

$$\sum_{i=1}^{n} w(k,i) = \frac{\sum_{i=1}^{n} w^+(i)}{m} \leq r_k. \quad (67)$$

Turning now to the set of weights $W(k,l)$ we have:

$$W^+(l) = r_l + \pi_l - \frac{\gamma_l}{q_*},\ 1 \leq l \leq m, \quad (68)$$

where:

$$q_* \geq \min_{1\leq l\leq m}[\frac{\gamma_l}{r_l + \pi_l}], \quad (69)$$

and for $1 \leq k \leq m$ we have:

$$W(l,k) = \frac{1}{m}[r_l + \pi_l - \frac{\gamma_l}{q_*}]. \quad (70)$$

XIII. Utility Functions and Optimisation

Simpler EPN optimisation problems, than the ones studied here were considered earlier in [52]. The optimisation would be conducted on the assumption that:

- The r_l are variable, but they have an upper bound that represents the maximum amount of power that can be delivered by the energy stores.
- The P_{ij} are fixed and represent the sequence of job steps that the jobs have to go through.
- The $S(l,j)$ and $s(l,k)$ are the "control variables": they are modified or selected so as to maximise U.
- Cases can be considered where the $S(l,k)$ are also fixed.

Obviously, we wish to limit the backlog of work at the workstations, while we also want to have some reserve of energy since there may be unpredictable needs. Thus a sensible cost or utility function would have the form:

$$U_1 = \sum_{i=1}^{n} a_i Q_i^{K_i} + \sum_{l=1}^{m} b_l (1 - q_l^{k_l}), \tag{71}$$

which is the weighted probability that the backlogs of work exceed the values K_i at each workstation i, and the weighted probabilities that there are **not** at least k_l energy packets at ES l, where the a_i and b_l are non-negative real numbers (the weights). A simpler utility that may be **minimised** is the weighted probabilities that there is some backlog at the workstations, plus that there is **no** energy in reserve, which is U_1 when $K_i = 1$ and $k_l = 1$ for all $i,\ l$:

$$U_1^0 = \sum_{i=1}^{n} a_i Q_i + \sum_{l=1}^{m} b_l (1 - q_l). \tag{72}$$

A cost function other than U_1 defined above that we may wish to minimise is:

$$U_2 = \sum_{i=1}^{n} a_i \frac{[\sum_{l=1}^{m} q_l w(l,i) + \beta_i]^{-1}}{1 - Q_i} + \sum_{l=1}^{m} b_l (1 - q_l^{k_l}), \tag{73}$$

which differs from U_1 only in the first sum which is simply the weighted sum of the average response time of jobs waiting to be served at each of the workstations.

On the other hand, the following utility function U_3 needs to be *maximised* since it establishes a balance between the throughput of the system (the first term) and the probability that some energy is kept in reserve:

$$U_3 = \sum_{l=1}^{m} [\sum_{i=1}^{n} a_i q_l w(l,i) Q_i + b_l q_l^{k_l}]. \tag{74}$$

XIV. Computing Partial Derivatives of the Utilities

The optimisation of the such utilities, where the utilities are continuous and differentialble functions of the control variables, will require the computation of the *derivatives* of the utility functions with respect to the control variables. In this case, the control variables are the routing probabilities $s(l,k)$ and $S(l,i)$ for the EPs. In fact because it is easier to work with non-negative real numbers of arbitrary size rather than with probabilities, we will consider that the control variables are the quantities $w(l,k)$ and $W(l,i)$.

Thus, for some real valued function V of the $w(l,i)$ and $W(k,l)$, $l,k \in \{1,\ \dots,m\}$ and $i \in \{1,\ \dots,n\}$, let us use the notation:

$$V^{w(l,i)} \equiv \frac{\partial V}{\partial w(l,i)}, \quad V^{W(k,l)} \equiv \frac{\partial V}{\partial W(k,l)}. \tag{75}$$

A. The Case where the r_l are constant

In a certain number of circumstances, we can consider that the r_l are constant, for instance when each of the ESs has a fixed and constant rate at which each of them can be emptied of its EPs. Here we will first focus on this case.

Remark 1 Since $r_l^{w(a,b)} = r_l^{W(a,b)} = 0$ for any $l,\ a,\ b$, it is easy to see that:

$$\begin{aligned} w(l,j)^{z(k,i)} &= 1,\ if\ 1[z=w,\ l=k,\ i=j]=1, \\ &\quad -1,\ if\ 1[z=w,\ l=k,\ i\neq j]=1, \\ &\quad -1,\ if\ 1[z=W,\ l=k,\ i\neq j]=1, \\ &\quad 0,\ if\ 1[z=w\ or\ W,\ l\neg k]=1, \end{aligned}$$

and

$$\begin{aligned} W(l,j)^{z(k,i)} &= 1,\ if\ 1[z=W,\ l=k,\ i=j], \\ &\quad -1,\ if\ 1[z=W,\ l=k,\ i\neq j]=1, \\ &\quad -1,\ if\ 1[z=w,\ l=k]=1, \\ &\quad 0,\ if\ 1[z=w\ or\ W,\ l\neq k]=1. \end{aligned}$$

As a direct application of Remark 1, we construct the following lemmas that will be used later in the optimising algorithms.

Lemma 1 The derivative of q_l with respect to any $w(a,b)$ for $l,\ a \in \{1,\ \dots,m\}$, $b \in \{1,\ \dots,n\}$ is given by:

$$q_l^{w(a,b)} = \frac{\sum_{k=1}^{m} q_k^{w(a,b)} W(k,l)\ - q_a}{r_l + \pi_l},$$

or in vector form:

$$q^{w(a,b)} = q^{w(a,b)}W - q_a\delta = q_a\delta[1-W]^{-1}, \tag{76}$$

where

- $q^{w(a,b)}$ is the m-vector each whose elements are the derivatives of the q_l,
- W is the $m \times m$ matrix of elements $W(l,k)/(r_k+\pi_k)$ and
- δ is the m-vector $\delta = [\frac{1}{r_1+\pi_1},\ \ldots\ ,\frac{1}{r_m+\pi_m}]$.

Lemma 2 The derivative of q_l with respect to any $W(a,b)$ for $l,\ a,\ b \in \{1,\ \ldots\ ,m\}$ is given by:

$$q_l^{W(a,b)} = \frac{\sum_{k=1}^m q_k^{W(a,b)}W(k,l)\ + q_a(1[l=b]-1[l\neq b])}{r_l+\pi_l},$$
$$q^{W(a,b)} = q_a.[2\delta_b - \delta][1-W]^{-1},$$

where $\delta_b = [0,\ \ldots\ ,0,\frac{1}{r_b+\pi_b},0,\ \ldots\ ,0]$ is the m-vector with 0 elements everywhere except in the b-th position which has the value $(r_b+\pi_b)^{-1}$.

Lemma 3 The derivative of Q_i with respect to any $w(a,b)$ for $a,\ b \in \{1,\ \ldots\ ,m\}$, $i,\ b \in \{1,\ \ldots\ ,n\}$ is given by:

$$Q_i^{w(a,b)} = \frac{\sum_{j=1}^n \sum_{l=1}^m [Q_j^{w(a,b)}q_l + Q_j q_l^{w(a,b)}]P_{ji}w(l,j)}{\sum_{l=1}^m q_l w(l,i)+\beta_i} + \frac{q_a[Q_bP_{bi} - \sum_{j\neq b,j=1}^n Q_jP_{ji}]}{\sum_{l=1}^m q_l w(l,i)+\beta_i} - Q_i\frac{\sum_{l=1}^m q_l^{w(a,b)}w(l,i) + q_a(1[b=i]-1[b\neq i])}{\sum_{l=1}^m q_l w(l,i)+\beta_i}$$

Lemma 4 The derivative of the Q_i with respect to any $W(a,b)$ for $a,b\ \in \{1,\ \ldots\ ,m\}$ is given by:

$$Q_i^{W(a,b)} = \frac{\sum_{j=1}^n \sum_{l=1}^m [Q_j^{W(a,b)}q_l + Q_j q_l^{W(a,b)}]P_{ji}w(l,j)}{\sum_{l=1}^m q_l w(l,i)+\beta_i} - \frac{\sum_{j=1}^n q_aQ_jP_{ji}}{\sum_{l=1}^m q_l w(l,i)+\beta_i} - Q_i\frac{\sum_{l=1}^m q_l^{W(a,b)}w(l,i) - q_a}{\sum_{l=1}^m q_l w(l,i)+\beta_i}$$

B. The Case where the r_l are not *constant*

In other circumstances the r_l will not be constant, so that when we modify any of the $w(l,i)$ or $W(l,k)$ we also change the r_l without affecting the other parameters.

This may occur for instance when the ESs are organised as a stack of batteries in parallel so that their output power flow can be varied as a function of the number of storage units that are switched to the power output bus.

However in practical circumstances there will be a maximuml value of $r_l^M \geq r_l$ which cannot be exceeded because of the physical limitations of the devices that are used to store energy and of the power circuits that carry the power flows.

Remark 2 Since $r_l^{w(a,b)} = r_l^{W(a,b)} = 0$ for any $l,\ a,\ b$, it is easy to see that:

$$\begin{aligned} w(l,j)^{z(k,i)} \quad = \quad & 1,\ if\ 1[z=w,\ l=k,\ i=j], \\ & -1,\ if\ z=w,\ l=k,\ i\neq j;\ r_l=r_l^M, \\ & -1,\ if\ z=W,\ l=k,\ i\neq j;\ r_l=r_l^M, \\ & 0,\ if\ z=w,\ l=k,\ i\neq j;\ r_l <_l^M \\ & 0,\ if\ z=W,\ l=k,\ i\neq j;\ r_l<r_l^M, \\ & 0,\ if\ z=w\ or\ W,\ l\neg k, \end{aligned}$$

and

$$\begin{aligned} W(l,j)^{z(k,i)} &= 1,\ if\ z=W,\ l=k,\ i=j, \\ &\quad -1,\ if\ z=W,\ l=k,\ i\neq j;\ r_l=r_l^M, \\ &\quad -1,\ if\ z=w,\ l=k,\ r_l=r_l^M, \\ &\quad 0,\ if\ z=W,\ l=k,\ i\neq j;\ r_l<r_l^M, \\ &\quad 0,\ if\ z=w,\ l=k,\ r_l=<_l^M, \\ &\quad 0,\ if\ z=w\ or\ W,\ l\neq k. \end{aligned}$$

As a consequence, when the $r_l < r_l^M$, $l = 1,\ \dots, m$ we have the following lemmas.

Lemma 5 The derivative of q_l with respect to any $w(a,b)$ for $l,\ a \in \{1,\ \dots, m\}$, $b \in \{1,\ \dots, n\}$ is given by:

$$\begin{aligned} q_l^{w(a,b)} &= \frac{\sum_{k=1}^m q_k^{w(a,b)} W(k,l)}{r_l+\pi_l} \\ &\quad -\frac{q_l}{r_l+\pi_l}, \end{aligned}$$

or in vector form:

$$q^{w(a,b)} = q^{w(a,b)}W - q1_l = -q1_l[1-W]^{-1}, \tag{77}$$

where as before W is the $m \times m$ matrix of elements $W(l,k)/(r_k+\pi_k)$, and 1_l is the $m \times m$ matrix which is zero everywhere except in the diagonal terms which have the value $\frac{1}{r_l+\pi_l}$.

Lemma 6 The derivative of q_l with respect to any $W(a,b)$ for $l,\ a,\ b \in \{1,\ \dots, m\}$ is given by:

$$\begin{aligned} q_l^{W(a,b)} &= \frac{\sum_{k=1}^m q_k^{W(a,b)} W(k,l) + q_a(1[l=b]-q_l)}{r_l+\pi_l}, \\ q^{W(a,b)} &= -[q_a\delta_b - q_l 1_l][1-W]^{-1}, \end{aligned}$$

where $\delta_b = [0,\ \dots, 0, \frac{1}{r_b+\pi_b}, 0,\ \dots, 0]$ is the m-vector with 0 elements everywhere except in the b-th position which has the value $(r_b+\pi_b)^{-1}$.

Lemma 7 The derivative of Q_i with respect to any $w(a,b)$ for $a,\ b \in \{1,\ \dots, m\}$, $i,\ b \in \{1,\ \dots, n\}$ is given by:

$$\begin{aligned} Q_i^{w(a,b)} = &\frac{\sum_{j=1}^n \sum_{l=1}^m [Q_j^{w(a,b)} q_l + Q_j q_l^{w(a,b)}] P_{ji} w(l,j)}{\sum_{l=1}^m q_l w(l,i) + \beta_i} \\ &+\frac{q_a Q_b P_{bi}}{\sum_{l=1}^m q_l w(l,i)+\beta_i} \\ &-Q_i \frac{\sum_{l=1}^m q_l^{w(a,b)} w(l,i) + q_a 1[i=b]}{\sum_{l=1}^m q_l w(l,i)+\beta_i} \end{aligned}$$

Lemma 8 The derivative of the Q_i with respect to any $W(a,b)$ for $a,\ b \in \{1,\ \dots, m\}$ is given by:

$$\begin{aligned} Q_i^{W(a,b)} = &\frac{\sum_{j=1}^n \sum_{l=1}^m [Q_j^{W(a,b)} q_l + Q_j q_l^{W(a,b)}] P_{ji} w(l,j)}{\sum_{l=1}^m q_l w(l,i) + \beta_i} \\ &-Q_i \frac{\sum_{l=1}^m q_l^{W(a,b)} w(l,i)}{\sum_{l=1}^m q_l w(l,i)+\beta_i}. \end{aligned}$$

XV. Optimal Solution with Gradient Descent

Since the optimisation of the EPN optimisation can be expressed as the minimization or maximisation of utility functions such as those defined in Section XIII, it can be achieved by selecting the appropriate EP flow rates $w(a,b)$ and $W(a,b)$ when other system parameters are fixed. Since we are delaing with continuous and differentiable utility functions, the gradient descent algorithm is a useful tool in this case.

At a given operating point of the EPN $X = [\lambda, \gamma, r, P, \pi, \beta]$, the gradient descent algorithm at its t^{th} computational step is:

$$w_{t+1}(a,b) = w_t(a,b) + \eta\ U^{w(a,b)}|_{w(a,b)=w_t(a,b)}, \tag{78}$$

$$W_{t+1}(a,b) = W_t(a,b) + \eta\ U^{W(a,b)}|_{W(a,b)=W_t(a,b)}, \tag{79}$$

where $|\eta|$ is the rate of the gradient descent, we set $\eta < 0$ fo rthe minimisation of utility functions U_1, U_1^0, U_2, while $\eta > 0$ forthe maximisation of utility function U_3.

In practice, we will be interested in *a gradual optimisation* of the system, where we modify parameters progressively, in a system that should operate normally and adapt the ongoing work work and energy flows.

Thus we compute the partial derivative of the utility functions as:

$$U_1^{w(a,b)} = \sum_{i=1}^{n} a_i K_i Q_i^{K_i-1} Q_i^{w(a,b)} - \sum_{l=1}^{m} b_l k_l (1 - q_l^{k_l-1}) q_l^{w(a,b)}, \tag{80}$$

similarly,

$$U_1^{0\,w(a,b)} = \sum_{i=1}^{n} a_i Q_i^{w(a,b)} - \sum_{l=1}^{m} b_l q_l^{w(a,b)}, \tag{81}$$

another cost function which we would like to minimise is:

$$\begin{aligned} U_2^{w(a,b)} = \sum_{i=1}^{n} a_i g(i) [\frac{1}{1-Q_i} Q_i^{w(a,b)} \\ - \frac{\sum_{l=1}^{m} (w(l,i) q_l^{w(a,b)} + q_l w(l,i)^{w(a,b)})}{[\sum_{l=1}^{m} q_l w(l,i) + \beta_i]}] \\ - \sum_{l=1}^{m} b_l k_l (1 - q_l^{k_l-1}) q_l^{w(a,b)}, \end{aligned} \tag{82}$$

where

$$g(i) = \frac{[\sum_{l=1}^{m} q_l w(l,i) + \beta_i]^{-1}}{1 - Q_i}. \tag{83}$$

Moreover, the partial derivative of the utility function which needs to be maximised is computed:

$$\begin{aligned} U_3^{w(a,b)} = \sum_{l=1}^{m} [\sum_{i=1}^{n} a_i [q_l^{w(a,b)} w(l,i) Q_i + q_l w(l,i)^{w(a,b)} Q_i \\ + q_l w(l,i) Q_i^{w(a,b)}] + b_l k_l q_l^{k_l-1} q_l^{w(a,b)}]. \end{aligned} \tag{84}$$

The partial derivative with respect to the $W(a,b)$ can be computed similarly. The $w(l,i)^{w(a,b)}$, $W(k,l)^{w(a,b)}$, $Q^{w(a,b)}$, $q^{w(a,b)}$, $w(l,i)^{W(a,b)}$, $W(k,l)^{W(a,b)}$, $Q^{W(a,b)}$, $q^{W(a,b)}$ are detailed, for both constant and variable r_l, in the previous section. Thus, the steps of the gradient algorithm are:

- First initialize all the values $w_0(a,b)$, $W_0(a,b)$ and choose η,
- Solve the system of non-linear equations given in (56) and (57) to obtain the steady state probabilities,
- Calculate the partial derivatives as given in Section XIV using the steady state probabilities computed in the previous step.
- Using these partial derivatives and the chain rule, compute the value of the relevant utility function from (80), (81), (82) or (84), and
- Finally update the values $w_{t+1}(a,b), W_{t+1}(a,b)$ according to (79).

XVI. A Set of Numerical Examples

To illustrate the proposed approach, we have constructed a numerical example for a facility, such as a remote sensing and monitoring facility such as an astronomical observatory on a mountain peak, or a border control station in a remote part of the world, that is powered by energy harvesting devices such as solar panels and wind turbines. The arrival rate to the workstations is in units, each of which can be executed or carried out with exactly one EP. Thus the relative workload for the different workstations is expressed by the arrival rate of work to each workstations. In the example we will consider there are three distinct workstations carrying out distinct work so that they can transfer work from one workstation to another and the $W(a,b) = 0$ for all $a \neq b$.

The work that uses energy is carried out in the three workstations:

- W_1 is meant to model a radio (or optical) telescope or radar station that is operating continuously and requires substantial processing, represented by a flow of work arriving at rate λ_1. We assign to it a priority $a_1 \geq 0$.
- W_2 represents the infrastructure energy needs such as lighting, air-conditioning etc., that receives work requests at rate λ_2, with $a_2 > 0$. Since this is a life-support system we would expect it to have high priority.
- W_3 is mean to model the perimeter security and surveillance, and the monitoring of weather conditions, with workload arriving at rate λ_3. Since this represents the life-support with a priority $a_3 > 0$, which may be comparable to that of a_2, while the priority a_1 may be lower.

Radars or telescopes provide a substantial amount of imaging data per unit time to be processed and transmitted, thus λ_1 can be considerably larger than λ_2 and λ_3. Moreover, security and atmospheric monitoring devices typically forward limited

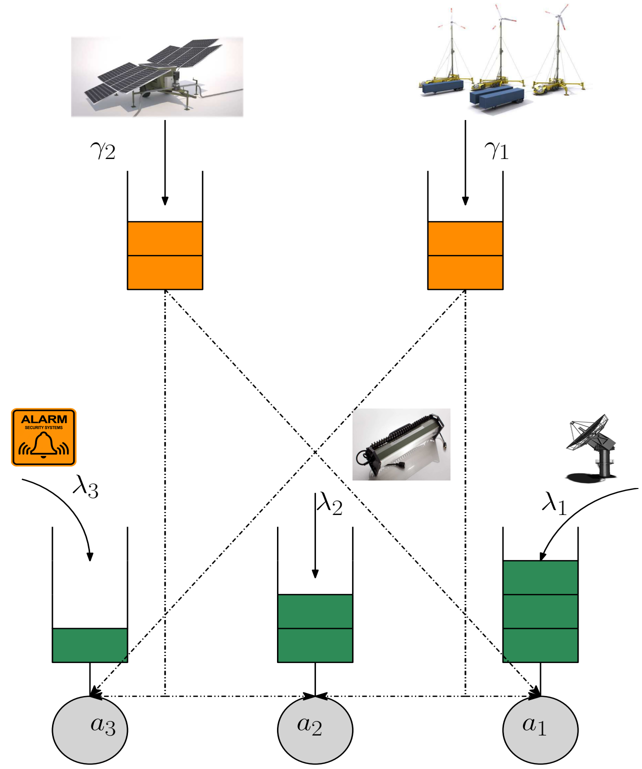

Fig. 2. We use the EPN to model the energy and work flows in a remotely located sensing facility, such as an astronomical observatory or radar station, with two energy storage units, two renewable energy sources (e.g. one photovoltaic and one wind) and three main energy consuming workstations: the major instrument (e.g. a radar), the life support systems (e.g. air conditioning, lighting), and the atmospheric sensing and perimeter security sensors.

TABLE I
PARAMETERS USED IN THE NUMERICAL EXAMPLES

Parameters	Values	Parameters	Values
λ_1	50	a_1	2
λ_2	20	a_2	1
λ_3	5	a_3	10
γ_1	40	β_1	6
γ_2	50	β_2	2
r_1^M	80	β_3	1
r_2^M	80	π_1, π_2	8
K_1, K_2, K_3	2	k_1, k_2	3

motion sensing, video camera, temperature and wind data, thus $\lambda_1 > \lambda_2 > \lambda_3$. Likewise, the priorities of these different work stations are chosento be $a_3 \geq a_2 > a_1$. To reflect different forms of energy harvesting, such as photovoltaic and wind, we have two ESs, E_1 and E_2 which store energy from two different renewable sources at a rate γ_1 and γ_2 EPs per unit time. The system model for this example is summarised in Figure 2. To obtain significant results and to remain within the stability region for the model, we must operate with parameters that represent a balance between the energy flow and the flow of workload. Also, the impatience of jobs, represented by the parameters β_i, can help avoid instability that occurs when the job queues increase in size and the waiting time of jobs at the execution queues become very large. Similarly, the energy leakage will also reduce the amount of energy that is stored when there is no work to perform. Note, however, that we will typically have $\pi_i << r_i^M$ to represent the fact that leakage rates are only a small fraction of the maximum power (energy/time) that the ESs can offer.

A plausible scenario is constructed with a set of numerical parameters shown in Table I in order to represent a plausible scenario. We first consider the case where the r_l values are not constant, and the case where they are constant.

A. When $r_1,\ r_2$ are not constant

Consider the case where the r_l values are not constant, but a maximum practical value r_l^M is chosen as in Table I. We set $P(a,b) = 0$ for $a,\ b \in \{1,2,3\}$ so that jobs are not moved between the different workstations. Also in the numerical examples we set $s(l,i) \geq 0.05\ , i = \{1,\ldots,3\},\ l = \{1,2\}$ and $\sum_{i=1}^{3} s(l,i) \leq 1, l = \{1,\ldots,2\}$ to make sure that each WS receives

TABLE II

OPTIMISED EP FLOW RATES FOR THE DIFFERENT UTILITY FUNCTIONS WHEN $b_1, b_2 = 100$, THE $r_1,\ r_2$ ARE NOT CONSTANT AND THE PARAMETERS ARE GIVEN IN TABLE I

Control parameters	Optimised rates for U_1^0	Optimised rates for U_1	Optimised rates for U_2	Optimised rates for U_3
$w^*(1,1)/r_1^M$	0.15	0.15	0.15	0.30
$w^*(1,2)/r_1^M$	0.20	0.20	0.20	0.05
$w^*(1,3)/r_1^M$	0.05	0.05	0.05	0.05
$w^*(2,1)/r_2^M$	0.45	0.45	0.45	0.30
$w^*(2,2)/r_2^M$	0.05	0.05	0.05	0.20
$w^*(2,3)/r_2^M$	0.05	0.05	0.05	0.05

TABLE III

OPTIMISED EP FLOW RATES FOR THE DIFFERENT UTILITY FUNCTIONS WHEN $b_1, b_2 = 10$, THE $r_1,\ r_2$ ARE NOT CONSTANT AND THE OTHER NUMERICAL PARAMETERS ARE IN TABLE I

Control parameters	Optimised rates for U_1^0	Optimised rates for U_1	Optimised rates for U_2	Optimised rates for U_3
$w^*(1,1)/r_1^M$	0.15	0.15	0.15	0.30
$w^*(1,2)/r_1^M$	0.15	0.20	0.20	0.05
$w^*(1,3)/r_1^M$	0.10	0.05	0.05	0.05
$w^*(2,1)/r_2^M$	0.45	0.45	0.45	0.30
$w^*(2,2)/r_2^M$	0.10	0.05	0.05	0.20
$w^*(2,3)/r_2^M$	0.05	0.05	0.05	0.05

some minimum amount of energy to avoid "starvation" that would lead to an infinite backlog of work at some workstations, and $\sum_{i=1}^{n} w(l,i) \leq r_l^M,\ l = \{1,\ldots,m\}$.

The unconstrained optimisation problem is solved numerically for the four utilities U_1, U_1^0, U_2, U_3 of Section XIII, and the resulting values for the optimum power flows are given in Tables II, III, IV, V for different b_1, b_2 values.

Table V shows that in order to maximise U_3 and minimise U_1, U_1^0, U_2, the ESs should be providing energy at their maximum total rate, i.e. $\sum_i w(l,i)^* = r_l^M$, when b_1, b_2 are considerably smaller than a_1, a_2, a_3. However when the b_1, b_2 are larger, we observe that the optima occur when the sum $\sum_i w(l,i)^*$ lies between $0.15 * r_l^M$ and r_l^M depending on the values of b_1, b_2. Note that the second terms in the utility functions concern the reserve energy contained in the ESs.

B. When $r_1,\ r_2$ are constant

When the r_1 and r_2 are constant, energy leaves each of the ESs at a constant rate. However not to waste energy, we will allow some of it to be sent to another ES rather than to the WS, if the optimisation requires it, i,e, the $P(a,b),\ a,b \in \{1,2\}$ are now unconstrained and can be positive.

This is illustrated in Tables , , , where we see that depending on the requirements of optimisation, a fraction of the energy sent out by the ESs can be forwarded for storage to the other ES. Note however than in this example, the energy loss during transfer, i.e. the ohmic loss, is not taken into account. Clearly, if energy transfer losses become significant then an optimised system will shy away from making additional energy transfers which do not lead to direct consumption.

TABLE IV

OPTIMISED EP FLOW RATES FOR THE DIFFERENT UTILITY FUNCTIONS WHEN $b_1, b_2 = 1$, THE $r_1,\ r_2$ ARE NOT CONSTANT AND THE OTHER NUMERICAL PARAMETERS ARE GIVEN IN TABLE I

Control parameters	Optimised rates for U_1^0	Optimised rates for U_1	Optimised rates for U_2	Optimised rates for U_3
$w^*(1,1)/r_1^M$	0.60	0.20	0.45	0.30
$w^*(1,2)/r_1^M$	0.20	0.15	0.45	0.40
$w^*(1,3)/r_1^M$	0.10	0.05	0.10	0.30
$w^*(2,1)/r_2^M$	0.40	0.45	0.40	0.45
$w^*(2,2)/r_2^M$	0.20	0.10	0.05	0.05
$w^*(2,3)/r_2^M$	0.30	0.15	0.10	0.05

TABLE V

OPTIMISED EP FLOW RATES FOR THE DIFFERENT UTILITY FUNCTIONS WHEN $b_1, b_2 = 0.1$, THE $r_1,\ r_2$ ARE NOT CONSTANT AND THE OTHER NUMERICAL PARAMETERS ARE GIVEN IN TABLE I

Control parameters	Optimised rates for U_1^0	Optimised rates for U_1	Optimised rates for U_2	Optimised rates for U_3
$w^*(1,1)/r_1^M$	0.40	0.85	0.45	0.60
$w^*(1,2)/r_1^M$	0.25	0.10	0.45	0.25
$w^*(1,3)/r_1^M$	0.35	0.05	0.10	0.15
$w^*(2,1)/r_2^M$	0.65	0.30	0.70	0.60
$w^*(2,2)/r_2^M$	0.20	0.35	0.10	0.20
$w^*(2,3)/r_2^M$	0.15	0.35	0.20	0.20

TABLE VI

OPTIMISED EP FLOW RATES FOR THE DIFFERENT UTILITY FUNCTIONS WHEN $b_1, b_2 = 100$, $r_1,\ r_2$ ARE CONSTANT AND THE OTHER NUMERICAL PARAMETERS ARE GIVEN IN TABLE I

Control parameters	Optimised rates for U_1^0	Optimised rates for U_1	Optimised rates for U_2	Optimised rates for U_3
$w^*(1,1)/r_1^M$	0.30	0.30	0.30	0.45
$w^*(1,2)/r_1^M$	0.20	0.20	0.20	0.05
$w^*(1,3)/r_1^M$	0.05	0.05	0.05	0.05
$w^*(2,1)/r_2^M$	0.30	0.30	0.30	0.15
$w^*(2,2)/r_2^M$	0.05	0.05	0.05	0.20
$w^*(2,3)/r_2^M$	0.05	0.05	0.05	0.05
$W^*(1,2)/r_1^M$	0.45	0.45	0.45	0.45
$W^*(2,1)/r_2^M$	0.60	0.60	0.60	0.60

TABLE VII

OPTIMISED EP FLOW RATES FOR THE DIFFERENT UTILITY FUNCTIONS WHEN $b_1, b_2 = 10$, $r_1,\ r_2$ ARE CONSTANT AND THE OTHER NUMERICAL PARAMETERS ARE GIVEN IN TABLE I

Control parameters	Optimised rates for U_1^0	Optimised rates for U_1	Optimised rates for U_2	Optimised rates for U_3
$w^*(1,1)/r_1^M$	0.40	0.30	0.30	0.45
$w^*(1,2)/r_1^M$	0.05	0.20	0.20	0.05
$w^*(1,3)/r_1^M$	0.05	0.05	0.05	0.05
$w^*(2,1)/r_2^M$	0.20	0.30	0.30	0.15
$w^*(2,2)/r_2^M$	0.20	0.05	0.05	0.20
$w^*(2,3)/r_2^M$	0.10	0.05	0.05	0.05
$W^*(1,2)/r_1^M$	0.50	0.45	0.45	0.45
$W^*(2,1)/r_2^M$	0.50	0.60	0.60	0.60

TABLE VIII

OPTIMISED EP FLOW RATES FOR THE DIFFERENT UTILITY FUNCTIONS WHEN $b_1, b_2 = 1$, $r_1,\ r_2$ ARE CONSTANT AND THE OTHER NUMERICAL PARAMETERS ARE GIVEN IN TABLE I

Control parameters	Optimised rates for U_1^0	Optimised rates for U_1	Optimised rates for U_2	Optimised rates for U_3
$w^*(1,1)/r_1^M$	0.60	0.20	0.10	0.05
$w^*(1,2)/r_1^M$	0.20	0.10	0.10	0.05
$w^*(1,3)/r_1^M$	0.10	0.15	0.05	0.05
$w^*(2,1)/r_2^M$	0.40	0.40	0.55	0.55
$w^*(2,2)/r_2^M$	0.20	0.15	0.20	0.20
$w^*(2,3)/r_2^M$	0.30	0.05	0.10	0.15
$W^*(1,2)/r_1^M$	0.1	0.55	0.75	0.85
$W^*(2,1)/r_2^M$	0.1	0.40	0.15	0.1

TABLE IX
OPTIMISED EP FLOW RATES FOR THE DIFFERENT UTILITY FUNCTIONS WHEN $b_1, b_2 = 0.1$, r_l ARE CONSTANT AND THE OTHER NUMERICAL PARAMETERS ARE GIVEN IN TABLE I

Control parameters	Optimised rates for U_1^0	Optimised rates for U_1	Optimised rates for U_2	Optimised rates for U_3
$w^*(1,1)/r_1^M$	0.60	0.55	0.25	0.70
$w^*(1,2)/r_1^M$	0.20	0.20	0.45	0.15
$w^*(1,3)/r_1^M$	0.10	0.20	0.25	0.05
$w^*(2,1)/r_2^M$	0.40	0.50	0.80	0.45
$w^*(2,2)/r_2^M$	0.20	0.25	0.05	0.25
$w^*(2,3)/r_2^M$	0.30	0.20	0.05	0.25
$W^*(1,2)/r_1^M$	0.1	0.05	0.05	0.1
$W^*(2,1)/r_2^M$	0.1	0.05	0.10	0.05

XVII. CONCLUSIONS

In this report, we have discussed the main properties of the Energy Packet Network model as a framework for a system where intermittent distributed workloads and intermittent sources of energy operate jointly with multiple energy consuming workstations, and with interconnected energy storage units, where both work and energy circulate. We obtain the equilibrium probability distribution for both the backlog of work and the backlog of energy throughout the system. An interesting special case is also considered where the work and energy flows have been designed so that all energy stores and work backlogs are the same. Several stochastic models that are more adapted to the study of wireless sensors with small scale energy harvesting [53], [17], [18], [6] have been also detailed elsewhere [54].

We have studied several utility functions that describe the overall system performance, and an algorithmic method based on gradient descent (or ascent) of these utility functions is developed to seek energy and work distribution mechanisms that optimise the utility functions. Also, the gradient descent approach we have described requires matrix inversions to compute $Q^{w(a,b)}$ and $q^{w(a,b)}$ which are of computational complexity $O(n^3)$ and $O(m^3)$, respectively. Simpler numerical schemes, such as power expansions of the matrices, can result in useful approximations, especially when the energy and work flow rates vary frequently with time, or are imprecisely known and will be studied in future work.

Another direction that could be considered includes more complex models in which a single job can consume more than one energy packet at a time, or where an energy packet may service several jobs at a time. The approach studied in this report can also be extended to multiple classes of EPs, with *power colouring* which can be used to represent the origin of the EPs, for instance different types of harvested energy from wind or photovoltaic, as well as from non-renewable or fossil sources, or with different economic cost.

Multiple classes of workloads depending on their energy needs, or their priorities, or their importance with regard to the income they produce, can also be considered. We will also examine "good" but suboptimal solutions which can be obtained at a lower computational cost. Finally, we will also study the dynamic behaviour of such systems when the flow of work and of energy is regulated in real-time, using techniques such as reinforcement learning.

REFERENCES

[1] J. Gubbi, R. Buyya, S. Marusic, and M. Palaniswami, "Internet of things (iot): A vision, architectural elements, and future directions," *Future Generation Computer Systems*, vol. 29, no. 7, pp. 1645 – 1660, 2013.

[2] A. Al-Ali and R. Aburukba, "Role of internet of things in the smart grid technology," *Journal of Computer and Communications*, vol. 3, pp. 229–233, 2015.

[3] E. Gelenbe and F.-J. Wu, "Large scale simulation for human evacuation and rescue," *Computers & Mathematics with Applications*, vol. 64, no. 12, pp. 3869–3880, 2012.

[4] T. McCue, "$117 billion market for internet of things in healthcare by 2020," *Forbes Tech*, April 2015. [Online]. Available: http://www.forbes.com/sites/tjmccue/2015/04/22/117-billion-market-for-internet-of-things-in-healthcare-by-2020/2715e4857a0b2536043d2471

[5] *Environmental Leader*, December 2015. [Online]. Available: http://www.environmentalleader.com/tag/internet-of-things

[6] E. Gelenbe, D. Gesbert, D. Gündüz, H. Külah, and E. Uysal-Biyikoglu, "Energy harvesting communication networks: optimization and demonstration: The e-crops project," in *Proc. 24th Tyrrhenian International Workshop 2013 on Digital Communications—Green ICT (TIWDC)*. Genoa, Italy: IEEE, 23–25 September 2013, pp. 1–6.

[7] E. Gelenbe and D. Gündüz, "Optimum power level for communications with interference," in *Proc. 24th Tyrrhenian International Workshop 2013 on Digital Communications—Green ICT (TIWDC)*. Genoa, Italy: IEEE, 23–25 September 2013, pp. 1–6.

[8] F. Akhtar and M. H. Rehmani, "Energy replenishment using renewable and traditional energy resources for sustainable wireless sensor networks: A review," *Elsevier Renewable and Sustainable Energy Reviews*, vol. 45, pp. 769–784, May 2015.

[9] A. Khan, M. Rehmani, and M. Reisslein, "Cognitive radio for smart grids: Survey of architectures, spectrum sensing mechanisms, and networking protocols," *IEEE Communications Surveys and Tutorials*, vol. 18, no. 1, pp. 860 – 898, 2015.

[10] R. C. Qiu, Z. Hu, Z. Chen, N. Guo, R. Ranganathan, S. Hou, and G. Zheng, "Cognitive radio network for the smart grid: Experimental system architecture, control algorithms, security, and microgrid testbed," *Smart Grid, IEEE Transactions on*, vol. 2, no. 4, pp. 724–740, 2011.

[11] Y. Zhang, R. Yu, M. Nekovee, Y. Liu, S. Xie, and S. Gjessing, "Cognitive machine-to-machine communications: visions and potentials for the smart grid," *Network, IEEE*, vol. 26, no. 3, pp. 6–13, 2012.

[12] E. Gelenbe and K. C. Sevcik, "Analysis of update synchronization for multiple copy data bases," *Transactions on Computers, IEEE*, vol. 28, no. 10, pp. 737–747, October 1979.

[13] A. des Technologies, *Impact des TIC sur la Consommation d'Énergie à travers le Monde*. Académie des Technologies (French National Academy of Technologies) & EDP Sciences, December 2015. [Online]. Available: http://www.academie-technologies.fr/blog/categories/rapports/posts/impact-des-tic-sur-la-consommation-d-energie-a-travers-le-monde

[14] E. Gelenbe, "A sensor node with energy harvesting," *SIGMETRICS Performance Evaluation Review*, vol. 42, 2014.

[15] E. Gelenbe and Y. Caseau, "The impact of information technology on energy consumption and carbon emissions," *Ubiquity*, no. June, pp. 1:1–1:15, 2015. [Online]. Available: http://doi.acm.org/10.1145/2755977

[16] B. Lannoo, S. Lambert, W. V. Heddeghem, and et al., "Overview of ict energy consumption (deliverable 8.1)," in *Report of the EU FP7 Project FP7-2888021*. Thessaloniki,Greece: European Network of Excellence in Internet Science, 2013.

[17] J. Yang and S. Ulukuş, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. Commun.*, vol. 60, pp. 220–230, 2012.

[18] A. Rahimi, O. Zorlu, and H. Külah, "A fully self-powered electromagnetic energy harvesting system with highly efficient dual rail output," *IEEE Sensours Journal*, vol. 12, pp. 2287–2298, 2012.

[19] E. Gelenbe and Y. M. Kadioglu, "Energy loss through standby and leakage in energy harvesting wireless sensors," in *20th IEEE International Workshop on Computer Aided Modelling and Design of Communication Links and Networks, CAMAD 2015, Guildford, United Kingdom, September 7-9, 2015*. IEEE, 2015, pp. 231–236. [Online]. Available: http://dx.doi.org/10.1109/CAMAD.2015.7390515

[20] I. Anghel, M. Bertoncini, T. Cioara, M. Cupelli, V. Georgiadou, P. Jahangiri, A. Monti, S. Murphy, A. Schoofs, and T. Velivassaki, "Geyser: Enabling green data centres in smart cities," in *Energy Efficient Data Centers*, ser. Lecture Notes in Computer Science, S. Klingert, M. Chinnici, and M. Rey Porto, Eds. Springer International Publishing, 2015, vol. 8945, pp. 71–86.

[21] M. He, E. Reutzel, X. Jiang, R. Katz, S. Sanders, D. Culler, and K. Lutz, "An architecture for local energy generation, distribution, and sharing," in *Energy 2030 Conference, 2008. ENERGY 2008. IEEE*, Nov 2008, pp. 1–6.

[22] B. Gurakan, O. Ozel, J. Yang, and S. Ulukus, "Energy cooperation in energy harvesting communications," *Communications, IEEE Transactions on*, vol. 61, no. 12, pp. 4884–4898, December 2013.

[23] K. Chandy, S. Low, U. Topcu, and H. Xu, "A simple optimal power flow model with energy storage," in *Decision and Control (CDC), 2010 49th IEEE Conference on*, Dec 2010, pp. 1051–1057.

[24] H.-I. Su and A. El Gamal, "Modeling and analysis of the role of fast-response energy storage in the smart grid," in *Communication, Control, and Computing (Allerton), 2011 49th Annual Allerton Conference on*, Sept 2011, pp. 719–726.

[25] E. Gelenbe, "Energy packet networks: Ict based energy allocation and storage - (invited paper)," in *GreeNets*, 2011, pp. 186–195.

[26] ——, "Energy packet networks: adaptive energy management for the cloud," in *Proceeding CloudCP '12, Proc. 2nd International Workshop on Cloud Computing Platforms*. ACM, 2012.

[27] ——, "Synchronising energy harvesting and data packets in a wireless sensor," *Energies*, vol. 8, no. 1, pp. 356–369, January 2015. [Online]. Available: www.mdpi.com/journal/energies

[28] ——, "Search in unknown random environments," *Phys. Rev. E*, vol. 82, p. 061112, 2010.

[29] J. Yang and S. Ulukuş, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. on Communications*, vol. 60, no. 1, pp. 220–230, January 2012.

[30] A. M. A. Rahimi, O. Zorlu and H. Külah, "A fully self-powered electromagnetic energy harvesting system with highly efficient dual rail output," *IEEE Sensors Journal*, vol. 12, no. 6, June 2012.

[31] S. Wang, J. Zhang, and L. Tong, "A characterization of delay performance of cognitive medium access," *Wireless Communications, IEEE Transactions on*, vol. 11, no. 2, pp. 800 –809, february 2012.

[32] D. Hamza and S. Aissa, "Impact of sensing errors on the queueing delay and transmit power in cognitive radio access," in *Communications and Information Technology (ICCIT), 2011 International Conference on*, march 2011, pp. 53 –58.

[33] J. Wang, A. Huang, W. Wang, and R. Yin, "Queueing analysis for cognitive radio networks with lower-layer considerations," in *Wireless Communications and Networking Conference (WCNC), 2012 IEEE*, april 2012, pp. 1269 –1274.

[34] D. Piazza, P. Cosman, L. Milstein, and G. Tartara, "Throughput and delay analysis for real-time applications in ad-hoc cognitive networks," in *Wireless Communications and Networking Conference (WCNC), 2010 IEEE*, april 2010, pp. 1–6.

[35] L.-C. Wang, C.-W. Wang, and F. Adachi, "Load-balancing spectrum decision for cognitive radio networks," *Selected Areas in Communications, IEEE Journal on*, vol. 29, no. 4, pp. 757–769, april 2011.

[36] L.-C. Wang, C.-W. Wang, and K.-T. Feng, "A queueing-theoretical framework for qos-enhanced spectrum management in cognitive radio networks," *Wireless Communications, IEEE*, vol. 18, no. 6, pp. 18–26, december 2011.

[37] R. Takahashi, T. Takuno, and T. Hikihara, "Estimation of power packet transfer properties on indoor power line channel," *Energies*, vol. 5, no. 7, pp. 2141–2149, 2012.

[38] C. Kim and A. Agrawala, "Analysis of the fork-join queue," *IEEE Transactions on Computers*, vol. 38, no. 2, pp. 250–255, 1989.

[39] E. Varki, "Mean value technique for closed fork-join networks," in *Proc. ACM SIGMETRICS Iinternational Conference on Measurement and Modeling of Computer Systems*, 1999, pp. 103–112.

[40] P. G. Harrison and S. Zertal, "Queueing models with maxima of service times," in *Proc. TOOLS Conference*, 2003, pp. 152–168.

[41] F. Baccelli, E. Gelenbe, and B. Plateau, "An end-to-end approach to the resequencing problem," *Journal of the ACM*, vol. 31, no. 3, pp. 474–485, 1984.

[42] E. Gelenbe, "Probabilistic models of computer systems part ii," *Acta Informatica*, vol. 12, pp. 285–303, 1979.

[43] ——, "The first decade of g-networks," *European Jouran of Operational Research*, vol. 126, no. 2, pp. 231–232, 2000.

[44] R. L. Tweedie, "Sufficient conditions for ergodicity and recurrence of markov chains on a general state space," *Stochastic Processes and their Applications*, vol. 3, no. 4, pp. 385–403, 1975.

[45] E. Gelenbe and A. Labed, "G-networks with multiple classes of signals and positive customers," *European Journal of Operational Research*, vol. 108, no. 2, pp. 293 – 305, 1998. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0377221797003718

[46] E. Gelenbe, "The first decade of g-networks," *EUROPEAN JOURNAL OF OPERATIONAL RESEARCH*, vol. 126, pp. 231–232, 2000. [Online]. Available: http://dx.doi.org/10.1016/S0377-2217(99)00475-0

[47] R. Takahashi, T. Takuno, and T. Hikihara, "Estimation of power packet transfer properties on indoor power line channel," *Energies*, vol. 5, no. 7, pp. 2141–2149, 2012.

[48] R. Takahashi, S.-I. Azuma, K. Tashiro, and T. Hikihara, "Design and experimental verification of power packet generation system for power packet dispatching system," in *American Control Conference (ACC), 2013*, June 2013, pp. 4368–4373.

[49] B. Kreiner and J. Reeves, "Packetized power," Jan. 9 2014, uS Patent App. 14/010,674. [Online]. Available: https://www.google.com/patents/US20140013146

[50] E. Gelenbe and E. T. Ceran, "Central or distributed energy storage for processors with energy harvesting," in *4th IFIP Conference on Sustainable Internet and ICT for Sustainability 2015: WiP and Demo session (SustainIT 2015 WiP and Demo Session)*, Madrid, Spain, Apr. 2015.

[51] E. Gelenbe, "G-networks with triggered customer movement," *Journal of Applied Probability*, vol. 30, no. 3, pp. 742–748, 1993. [Online]. Available: http://www.jstor.org/stable/3214781

[52] E. Gelenbe and C. Morfopoulou, "A framework for energy-aware routing in packet networks," *The Computer Journal*, vol. 54, no. 6, pp. 850–859, 2011.

[53] O. Orhan, D. Gunduz, and E. Erkip, "Throughput maximization for an energy harvesting communication system with processing cost," in *IEEE Information Theory Workshop (ITW), Lausanne, Switzerland*, 2012.

[54] E. Gelenbe and A. Marin, "Interconnected wireless sensors with energy harvesting," in *Analytical and Stochastic Modelling Techniques and Applications*, ser. Lecture Notes in Computer Science, M. Gribaudo, D. Manini, and A. Remke, Eds. Springer International Publishing, 2015, vol. 9081, pp. 87–99.