Search in unknown random environments

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N searchers are sent out by a source in order to locate a fixed object which is at a finite distance *D*, but the search space is infinite and *D* would be in general unknown. Each of the searchers has a finite random lifetime, and may be subject to destruction or failures, and it moves independently of other searchers, and at intermediate locations some partial random information may be available about which way to go. When a searcher is destroyed or disabled, or when it "dies naturally," after some time the source becomes aware of this and it sends out another searcher, which proceeds similarly to the one that it replaces. The search ends when one of the searchers finds the object being sought. We use *N* coupled Brownian motions to derive a closed form expression for the average search time as a function of *D* which will depend on the parameters of the problem: the number of searchers, the average lifetime of searchers, the routing uncertainty, and the failure or destruction rate of searchers. We also examine the cost in terms of the total energy that is expended in the search.

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I. INTRODUCTION

It is very common to search for a known or recognizable object, without knowing the path to the object in a precise manner, or knowing only imprecise information. Once the searcher is close to the object being sought, it can detect or recognize it (e.g., smell generated by food), however the challenge is to get close enough to it without exact information about where it is. If the search space is infinite, the searcher may also become lost "forever." The search process may be dangerous for the searcher and it may be destroyed (e.g., eaten by a predator). The searcher will often have a finite life span (e.g., finite fuel for a mobile robot), which is known at least in probabilistic terms. Examples of such situations include: (i) foraging by an animal that can recognize an edible object or a mate when it finds it, but does not know exactly where to find it, and can also be harmed during the search, (ii) packets traveling in a very large wireless or wired network [1,2], without the benefit of fully reliable routing tables in intermediate network nodes, with possible packet loss due to transmission errors or buffer overflows, (iii) computer search of specific data or a complex digitally represented object (which may be a visual entity) in a very large number database [3], with a finite computational budget (life span), and the possibility that the software may fail to run in some part of the database, (iv) motion of a particle under the effect of a random field; in this case the "object being sought" may be a location with an opposite electric charge that ends the movement of the "searcher" particle: the finite life span can result from the decay of the searcher's charge, (v) motion of a biological agent [4] until it docks onto a specific site where it can become active, while it loses its reactivity as it ages, (vi) motion of a physical robot sent to find a specific object with a finite reserve of fuel. For instance, packets in an *ad hoc* wireless network travel over a random number of relay nodes toward a destination whose location may not be precisely known [5]. In such systems once a packet reaches its destination it may be able to send back an acknowledgment by reversing the path it used and avoiding any repetitions in the nodes visited. If the source has not heard back from the packet after some predetermined time (the "time out"), the sender sends out another packet on the assumption that the previous packet has been lost; if the current packet is not lost or dead, it will self-destroy at the same time out to avoid having duplicate packets in the network [6,7].

The work in the present paper is primarily motivated by (ii) and (iv) above, in that the departure point of all the searchers is exactly the same, so that their distance to the object being sought is an identical quantity *D* for all of the searchers.

Starting from different perspectives, several authors have analyzed such systems with different physical assumptions. For instance, the work in [8,9] models the search behavior of an animal which replenishes its energy supply while it forages and searches; energy dissipation and replenishment are judiciously included in the Langevin equation used to represent the search process, and an approximate solution is obtained. In [10] the search space is represented by a sequence of finite graphs with probabilistic connection, as one may represent a wired computer network or a system of roads, and detailed first passage time probabilities are derived for a random initial search location. Search for a prey which will jump away at random when the predator gets close is considered in [11], and interesting results are derived for the number of searchers that are needed to guarantee that the prey is caught in a finite search area. In [12] the search is conducted as a random walk with jumps, so that one probabilistically alternates neighborhood movement with random jumps; interesting results are derived about how this alternate motion can be conducted to optimize the search.

Simulation examples

Before we proceed with our analysis, we will present two simulation examples within the framework of our approach. In the case that we consider, the search space is infinite, i.e.,

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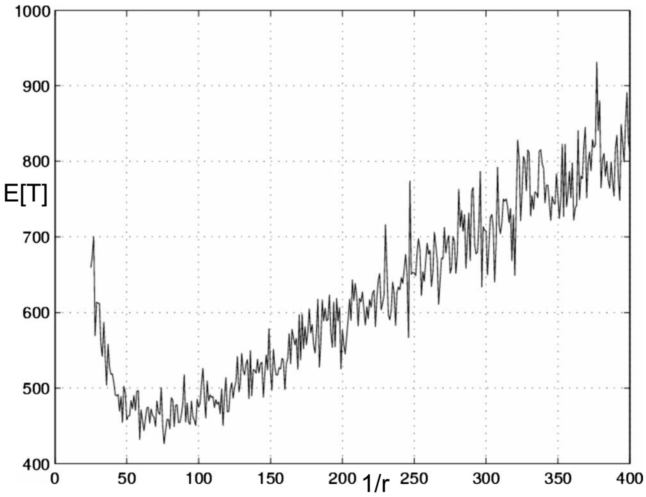

FIG. 1. Monte Carlo simulation of the average search time $E[T^*]$ until the object sought is found by a single search agent or packet $N=1$; the x axis is the average time-out value of the time-out r^{-1}. The packet travels reliably (so that it cannot be destroyed other than at the time out), and from any location it can reach any neighboring location North-East-South-West in one time unit. All nodes operate under "perfect ignorance" ($b=0$) so that the packet is equally likely to get further or closer to the destination in one hop. The total distance between the starting point of the search and the location of the object that is sought is $D=10$. After the time out, the sender at the origin where the search initiates will wait on average ten more time units ($\mu=0.1$), before sending out another packet. Each point on the curve is the average of 20 simulation runs with the same parameter set.

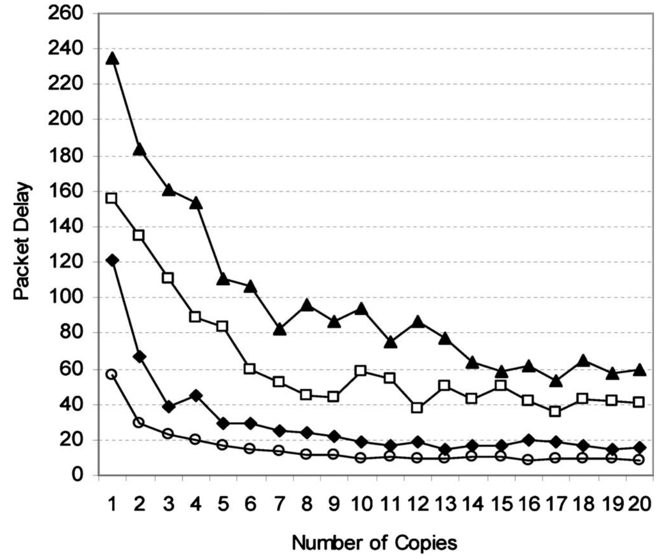

FIG. 2. Monte Carlo simulation results for the average travel time from source to destination for the first among N packets to arrive at the destination, versus N the number of packets that are simultaneously sent. The initial distance is $D=50$, and the average time out is $r^{-1}=600$. Each point on the curve is the average of 40 simulation runs. The intermediate transceivers are placed in a regular rectangular unit grid, and the one time step transmission range of a packet is varied from 6 (top curve) to 16 (bottom curve) hops.

it has no natural finite boundaries that limit the area/volume in which the search is conducted so that a searcher may meander indefinitely and still not find the object being sought. However, the object being sought is at a finite distance D from the initial point of the search, but D is unknown. Furthermore, each searcher has a probabilistically finite lifetime, but after this lifetime or "time out" elapses a new searcher will be sent out from the initial point so that the search can be repeated again.

In the first simulation example, the average travel time of a single packet that is sent out toward an unknown destination in a two-dimensional grid of wireless transceivers is shown in Fig. 1 using Monte Carlo simulations, as a function of the average value of the finite lifetime or time out of the packet. Here, the packet may be picked up by any one of the immediate neighboring nodes after one step. The simulation results clearly show the strong influence of the time-out parameter r on the average overall time it takes the packet to find its destination, where r^{-1} is the average value of the time out.

The next simulations consider the effect of the number N of duplicate packets or searchers that are simultaneously sent out, as well as on the distance that a packet can travel in one hop. Figure 2 summarizes results from Monte Carlo simulations for the average travel time of the first, among N packets sent simultaneously from the same source, that reaches the destination node in a regularly spaced two-dimensional grid of wireless transceivers. Each point on the curves is the average earliest arrival time among the N packets, for 40 simulations conducted in identical circumstances, and we clearly see how increasing the number of searchers and also increasing the travel distance in one hop will reduce the average search time. The top curve corresponds to the case when the travel distance in one hop for the packet is six units, while the following curves that are below it correspond to 10, 14, 16 units of distance, respectively.

II. MODELING THE SEARCH PROCESS

Let the search begin at time $t=0$ and number the searchers from 1 to N. Let $Y_i(t)$ denotes the ith searcher's distance from its destination at time $t \geq 0$; each searcher starts at distance $Y_i(0)=D$. The effective search time $T^*(N)$ is then obtained from the variables,

$$T_i = \inf\{t : Y_i(t) = 0\}, \quad (1)$$

so that $T^*(N)=\inf\{T_1, \ldots, T_N\}$. Let $s_i(t)$ denote the state of the searcher at time $t \geq 0$. Then $s_i(t)$ can take one of the values $\{\mathbf{S_i}, \mathbf{L_i}, \mathbf{W_i}, \mathbf{P}\}$ which are defined as follows:

(i) $\mathbf{S_i}$: if the ith searcher's search is going on and its position from the destination is $Y_i(t) > 0$. We denote the probability density function of the position $Y_i(t)$ by $f_i(z_i,t)dz_i = P[z_i < Y_i(t) \leq z_i + dz_i \,\&\, s_i(t) = \mathbf{S_i}]$.

(ii) $\mathbf{L_i}$: it has been destroyed or lost, and its search is ended. The time spent in this state is exponentially distributed with the same parameter r as the life span since the source realizes that the searcher is lost or destroyed via the life-span effect. At the end of this exponentially distributed time, the searcher is handled just as if it has "died" (see the next point). We write $L_i(t) = P[s_i(t) = \mathbf{L_i}]$.

(iii) $\mathbf{W_i}$: its life span has ended, and so has its search. Note that this may have happened (previous point) because it was destroyed or became lost, but this becomes known to the source via the life-span effect. After an additional exponentially distributed delay of parameter μ, meant to avoid mistakes in assuming that the ith searcher is "dead," it is replaced at the source by a new searcher with the same identity. We write $W_i(t)=P[s_i(t)=\mathbf{W_i}]$.

(iv) $\mathbf{P}$: One of the searchers has found the object being sought; the search process stops for all searchers, including the ones who are lost or dead. Notice that $\mathbf{P}$ is a synchronized state for all of the searchers. After one time unit, the search process starts again as before at the source with N searchers being sent out. We write $P(t)=P[s_i(t)=\mathbf{P}]$.

Notice that the above process repeats itself indefinitely, and $E[T^*]$ is the average time that it takes from any successive start of the search until the first instance when state $\mathbf{P}$ is reached again. Let $P(t)$ be the probability that the model we have just described is in state $\mathbf{P}$ at time $t \geq 0$, and let $P=\lim_{t\to\infty} P(t)$. Then

$$P = \frac{1}{1+E[T^*]}, \quad E[T^*] = \frac{1-P}{P}. \tag{2}$$

During the ith searcher's travel in state $\mathbf{S_i}$ while $\{Y_i(t)=y>0\}$ the following events can occur in the time interval $[t,t+\Delta t[$:

(i) With probability $\lambda\Delta t+o(\Delta t)$ the ith searcher is destroyed or lost, and enters state $\mathbf{L_i}$. From that state it enters state $\mathbf{W_i}$ after an exponentially distributed delay of parameter.

(ii) With probability $r\Delta t+o(\Delta t)$ the searcher's life span runs out and it enters state $\mathbf{W_i}$. Note that $\frac{1}{r}$ is the average life span. As indicated earlier, when it enters state $\mathbf{W_i}$, after an additional delay of average value $\frac{1}{\mu}$, the ith searcher is replaced with a new one at the source.

A real number b represents the average rate of change over time $[t,t+\Delta t[$ of the searcher's distance to the destination, and the variance of the distance traveled by the searcher in that time interval is $c\Delta t$, $c \geq 0$,

$$b = \lim_{\Delta t\to 0} \frac{E[Y_{t+\Delta t}-Y_t|Y_t=y]}{\Delta t},$$

$$c = \lim_{\Delta t\to 0} \frac{E[(Y_{t+\Delta t}-Y_t)^2]-(E[Y_{t+\Delta t}-Y_t])^2|Y_t=y]}{\Delta t},$$

so that we assume that the medium in which the searchers move is homogenous in space and time. While $b<0$ is the favorable case where the searcher on average gets closer to the destination with time, we may also have cases of interest with $b>0$, which means that the searcher on average moves *away* from the object of interest, for instance because intermediate locations provide wrong information on average, or it lacks information altogether when $b=0$. It was shown that even when $b \geq 0$ it is possible to have a travel time to destination which is finite on average [7].

We now express the process $\{s_i(t):t\geq 0\}$ in terms of a system of equations describing a somewhat unusual mixed continuous space (diffusion) and discrete space random process [13–17]. We first write the equations that the probability density function $f_i(z_i,t)dz_i$, $z_i>0$, and the probability masses $L_i(t)$, $W_i(t)$ and $P(t)$, $t\geq 0$ will satisfy.

We represent the interaction between the diffusion processes using the parameter $a_i(t)$, $1\leq i\leq N$ in the following unusual manner; $a_i(t)$ is the total rate of attraction exerted at time t by all other diffusion processes, on the ith diffusion due to the fact that one of the other diffusions may have reached its absorbing barrier at $z_j=0$ to represent the event when the jth searcher has found the object being sought. The system of coupled differential and partial differential equations representing the search are

$$\frac{\partial f_i(z_i,t)}{\partial t} = -[\lambda + r + a_i(t)]f_i(z_i,t) - b\frac{\partial f_i(z_i,t)}{\partial z_i} + \frac{1}{2}c\frac{\partial^2 f_i(z_i,t)}{\partial z_i^2} + [P(t) + \mu W_i(t)]\delta(z_i - D), \tag{3}$$

while

$$\frac{dL_i(t)}{dt} = -[r + a_i(t)]L_i(t) + \lambda\int_{0^+}^{\infty} f_i(z_i,t)dz_i, \tag{4}$$

$$\frac{dW_i(t)}{dt} = -[\mu + a_i(t)]W_i(t) + r\left[L_i(t) + \int_{0^+}^{\infty} f_i(z_i,t)dz_i\right], \tag{5}$$

$$\frac{dP(t)}{dt} = -P(t) + \sum_{i=1}^{N}\lim_{z_i\to 0^+}\left[-bf_i(z_i,t) + \frac{1}{2}c\frac{\partial f_i(z_i,t)}{\partial z_i}\right], \tag{6}$$

and

$$a_i(t) = \sum_{j=1,j\neq i}^{N}\lim_{z_j\to 0^+}\left[-bf_j(z_j,t) + \frac{1}{2}c\frac{\partial f_j(z_i,t)}{\partial z_j}\right]. \tag{7}$$

We also have that the sum of the probabilities is one.

$$1 = L_i(t) + W_i(t) + P(t) + \int_{0^+}^{\infty} f_i(z_i,t)dz_i. \tag{8}$$

From Eq. (7) we see that $a_i(t)$ is the rate at which the ith searcher is attracted to the origin, i.e., to finish its search, because any one of the *other* $N-1$ searchers has found the object being sought.

(i) This is reflected both in Eq. (4) and in the Eqs. (5) and (6) where the searcher can be forced into the rest state from the "lost" state and the "time out before retransmission" state, as well. We also see that we enter the loss state from any position $z_i>0$, and that a time out can occur for a searcher that is in the lost state.

(ii) Since the behavior of all searchers when they are not in the rest state are independent, it follows that the event that triggers the jump of searcher i into the rest state does not depend on the prior state of searcher i but on the state of the *other* searchers.

(iii) In Eq. (4) we can see the terms related to the rate of loss λ and the time-out rate r, as well as the jump back to the start state both from the rest state and the time-out state.

Note again that these equations represent the system where, whenever any one searcher has reached the destination, all other searchers' progress is artificially stopped and restarted from the rest state s. The purpose here is to compute $E[T^*]$ by constructing a synthetic ergodic process.

The system of differential and partial derivative equations for $1 \leq iN$ takes the following form in steady-state:

$$0 = -(\lambda + r + a_i)f_i(z_i) - b\frac{\partial f_i(z_i)}{\partial z_i} + \frac{1}{2}c\frac{\partial^2 f_i(z_i)}{\partial z_i^2} + [P + \mu W_i]\delta(z_i - D), \quad (9)$$

while

$$(r + a_i)L_i = \lambda \int_{0^+}^{\infty} f_i(z_i)dz_i, \quad (10)$$

$$(\mu + a_i)W_i = r\left(L_i + \int_{0^+}^{\infty} f_i(z_i)dz_i\right), \quad (11)$$

$$P = \sum_{i=1}^{N} \lim_{z_i \to 0^+}\left[bf_i(z_i) + \frac{1}{2}c\frac{\partial f_i(z_i)}{\partial z_i}\right], \quad (12)$$

$$a_i = \sum_{j=1, j\neq i}^{N} \lim_{z_j \to 0^+}\left[-bf_j(z_j) + \frac{1}{2}c\frac{\partial f_j(z_i)}{\partial z_j}\right], \quad (13)$$

with

$$1 = L_i + W_i + P + \int_{0^+}^{\infty} f_i(z_i)dz_i. \quad (14)$$

Dropping the dependence on i because all searchers are statistically identical, we write the characteristic polynomial of the diffusion equation,

$$0 = -(\lambda + r + a) - bu + \frac{1}{2}cu^2, \quad (15)$$

which has two real roots,

$$u_1, u_2 = \frac{b \pm \sqrt{b^2 + 2c(\lambda + r + a_i)}}{c}. \quad (16)$$

Note that one root is always non-negative, both when b is positive or negative. Since we are seeking a solution which is a probability density function whose integral over $[0, +\infty]$ must be finite, for $z \geq D$ we can only use the negative root $u_2 = \frac{b - \sqrt{b^2 + 2c(\lambda + r + a_i)}}{c}$, while when $z < D$ we use both roots,

$$f(z) = Ce^{u_2 z}, \quad z \geq D,$$

$$f(z) = Ae^{u_1 z} + Be^{u_2 z} + F, \quad 0 \leq z_i < D,$$

where the $A, B, C, F \geq 0$ are constants. Because $f(z,t)$ has an absorbing boundary at $z=0$ and $f(0)=0$, we get $F=-(A+B)$. Furthermore, using Eq. (9) at $z=0$ we have

$$b[u_1 A + u_2 B] = \frac{1}{2}c[Au_1^2 + Bu_2^2], \quad (17)$$

which results in $B=-A$ so that we end up with

$$f(z) = A[e^{u_1 z} - e^{u_2 z}], \quad 0 \leq z < D. \quad (18)$$

Using the continuity at $z=D$ we have $Ce^{u_2 D} = A[e^{u_1 D} - e^{u_2 D}]$ so that

$$f(z) = A[e^{(u_1 - u_2)D} - 1]e^{u_2 z}, \quad z \geq D, \quad (19)$$

Denote $Q = \int_{0^+}^{\infty} f_{(z)dz}$ so that

$$Q = A[e^{u_1 D} - 1]\left[\frac{1}{u_1} - \frac{1}{u_2}\right],$$

$$= A[e^{u_1 D} - 1]\frac{\sqrt{b^2 + 2c(\lambda + r + a)}}{\lambda + r + a}, \quad (20)$$

and using Eq. (20) with Eq. (14), and Eqs. (11)–(13) we end up with

$$A = \left\{\sqrt{b^2 + 2c(\lambda + r + a)} \times \left[N + \frac{e^{u_1 D} - 1}{\lambda + r + a}\left(1 + \frac{\lambda}{r + a}\right)\left(1 + \frac{r}{\mu + a}\right)\right]\right\}^{-1}, \quad (21)$$

$$a = \frac{1}{2}Ac(N-1)[u_1 - u_2] = (N-1)A\sqrt{b^2 + 2c(\lambda + r + a)}. \quad (22)$$

Thus using Eqs. (21) and (22) we can compute A and a, and $P = AN\sqrt{b^2 + 2c(\lambda + r + a)}$ so that

$$P = \frac{N}{N + \frac{e^{u_1 D} - 1}{\lambda + r + a}\left(1 + \frac{\lambda}{r + a}\right)\left(1 + \frac{r}{\mu + a}\right)},$$

$$E[T^*] = [AN\sqrt{b^2 + 2c(\lambda + r + a)}]^{-1} - 1,$$

$$= \frac{e^{u_1 D} - 1}{N(\lambda + r + a)}\left(1 + \frac{\lambda}{r + a}\right)\left(1 + \frac{r}{\mu + a}\right)$$

$$= \frac{e^{D/c[b + \sqrt{b^2 + 2c(\lambda + r + a)}]} - 1}{N} \cdot \frac{(\mu + r + a)}{(r + a)(\mu + a)}, \quad (23)$$

which is easier to interpret when we multiply both the numerator and denominator of the exponent by u_2, yielding

$$E[T^*] = [e^{-2D(\lambda + r + a/b - \sqrt{b^2 + 2c(\lambda + r + a)})} - 1]\frac{(\mu + r + a)}{N(r + a)(\mu + a)}. \quad (24)$$

A. Simulation and numerical examples

In Fig. 3 the theoretical prediction (the solid line) from Eq. (24) is compared with a Monte Carlo simulation for the case $N=1$. It appears that the theory estimates a larger average search time than the simulation; this may be due, especially when the search times are shorter around the optimum

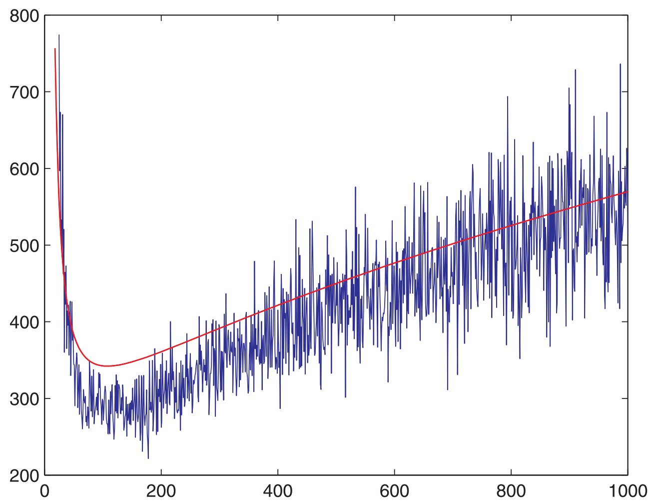

FIG. 3. (Color online) The average search time $E[T^*]$ until the object is found by a single searcher ($N=1$) is plotted using the theoretical analysis (solid line) and Monte Carlo simulations on a rectangular unit grid. There are no losses $\lambda=0$, and $D=10$, $b=0.5$, $c=1$, $\mu=0.1$. Each simulation point on the curve is the average of 20 simulation runs with the same parameter set.

value of $1/r$, to the slower convergence of the discrete event simulations to the continuous Brownian motion when the times are shorter. The next numerical examples are based on the predictions of the theory. Figure 4 illustrates the effect of c and N as $1/r$ varies: we see that more randomness, i.e., a larger c, reduces the search times, and that N has a substantial effect especially for smaller c. Figure 5 shows that a higher searcher loss rate λ will substantially lengthen the average search time, and that this can be compensated with larger N.

Figure 6 shows how the average search time varies with N for various values of b; we see that N has a particularly strong effect when $b>0$, i.e., when at each intermediate step, the information available is sending the searchers on average *away from the object being searched*. Finally in Fig. 7, again

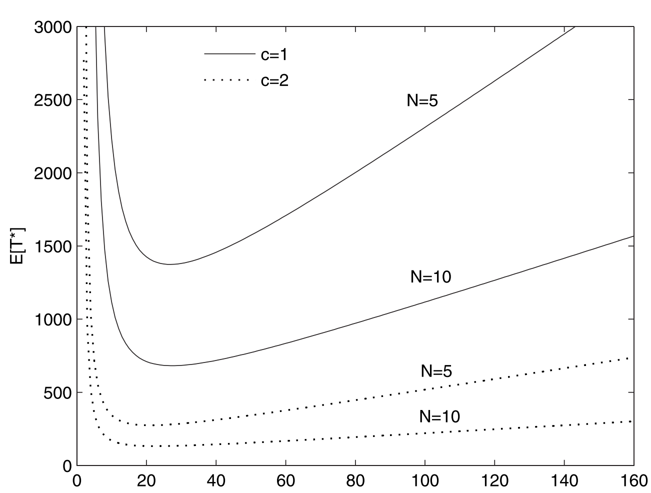

FIG. 4. Average search time to find the object versus the average time out $1/r$ for $D=10, \mu=0.1, \lambda=0.1$ and $b=0$ and different values of N.

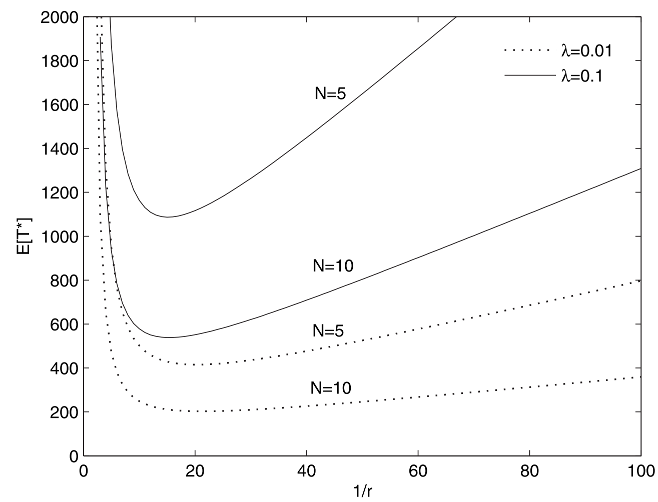

FIG. 5. Average search time to find the object versus the average time out $1/r$ for $D=10, \mu=0.1, c=3$ and $b=0.5$ with varying N.

for $b>0$, we illustrate the effect of N on the average search time for different values of c and of the loss rate λ. Higher loss rates, and less randomness in the search, i.e., smaller c, will increase the average search time for all values of N.

B. When *D* is a random variable

Note that Eqs. (21) and (22) allow us to compute a as a function of A quite easily, but A has a nonlinear dependence on a and D. Thus we do not expect that we can obtain a simple closed form expression for a as a function of D, and hence we do not expect to be able to compute $E[T^*]$ as a function of D in some general simple explicit form. Our purpose in this paper is to ask how the average search time varies with D and N, as well as the other parameters of the problem, but essentially the results have to be computed numerically from the nonlinear dependence of Eq. (21) together with the simpler expression (22). Therefore more generally we do not expect that we can derive closed-form expressions

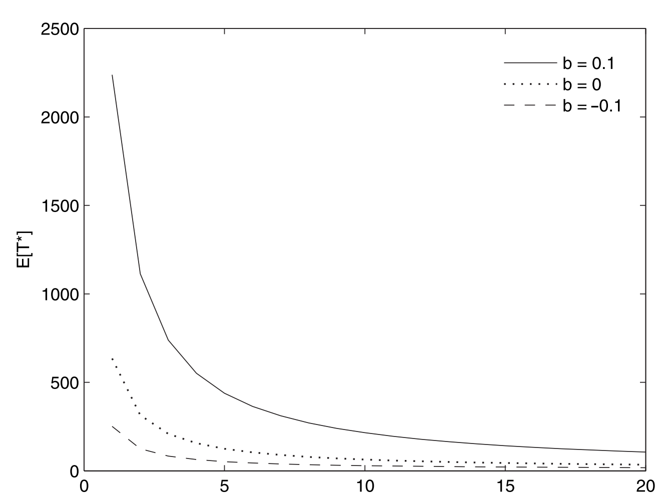

FIG. 6. Average search time to find the object versus the number of searchers N with $D=10$, $\mu=0.1$, $c=1$, $\lambda=0.01$, and $r=0.02$.

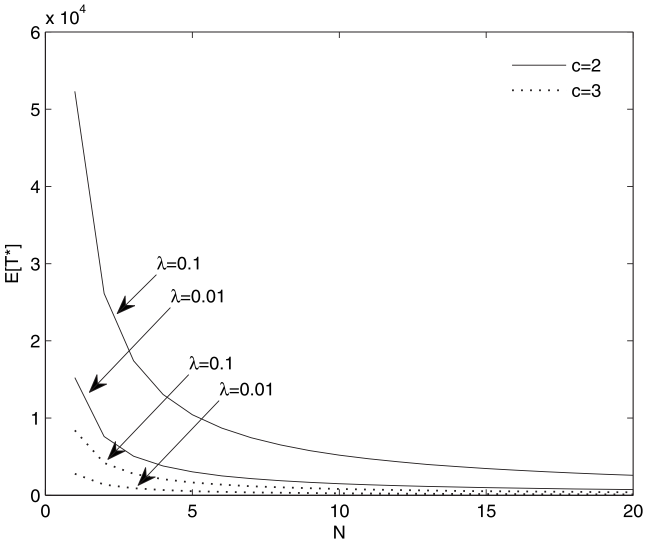

FIG. 7. Average search time to find the object versus the number of searchers N with $D=10$, $\mu=0.1$, $b=0.5$, and $r=0.02$.

for $E[T^*]$ when D is a random variable. However, this can be done at least in the simple case when $N=1$ and hence $a=0$.

If D is a random variable and $N=1$ we have

$$E[T^*] = [e^{-2D(\lambda+r/b-\sqrt{b^2+2c(\lambda+r)})} - 1]\left(\frac{1}{r}+\frac{1}{\mu}\right). \quad (25)$$

If D is expressed in the form of a probability density function $g(D)$, the analysis tells us that the random search by a single searcher will find the object in an average time,

$$\langle T\rangle = \int_0^\infty E(T^*)g(D)dD = \left[g^*\left\{2D\left[\frac{\lambda+r}{b-\sqrt{b^2+2c(\lambda+r)}}\right]-1\right]\left(\frac{1}{r}+\frac{1}{\mu}\right), \quad (26)$$

where $d^*(s)$ is the Laplace-Stietjes transform of $g(D)$.

For instance, assuming "perfect ignorance" with $b=0$, and taking $\lambda=0$ so that a searcher cannot be disabled during the search process, we have

$$\langle T\rangle = \int_0^\infty E[T^*]g(D)dD = \left[g^*\left(-\sqrt{\frac{2r}{c}}\right)-1\right]\left(\frac{1}{r}+\frac{1}{\mu}\right). \quad (27)$$

If we consider the case where $g(D)=\frac{1}{E[D]}e^{-D/E[D]}$, i.e., the exponential distribution with mean $E[D]$, we obtain

$$\langle T\rangle = \frac{1}{1-E[D]\sqrt{\frac{2r}{c}}}\left(\frac{1}{r}+\frac{1}{\mu}\right)$$

and we see that in this case the average search time is finite only if r is small enough so that $E[D]<\sqrt{\frac{c}{2r}}$. Thus the designer of the search strategy would try to select a time-out value that is big enough in relation to the remaining characteristic parameters of the search, i.e.,

$$\frac{1}{r} > \frac{2(E[D])^2}{c}.$$

III. ENERGY CONSUMPTION

If the search is carried out by a physically moving searcher such as a robot, the energy consumed will depend on its velocity and positive acceleration (while deceleration can potentially be used to store energy). In a virtual search, the speed of computation (and hence the rate at which the search progresses over time) can also affect energy consumption with higher speeds costing more energy. In wireless transmission things are more complicated because higher transmission speeds may be more or less error prone, or prone to interference or collisions with other communications, depending on the frequency bands that are used and on the time during which the channel is occupied by the transmission.

Here we simplify matters and assume that a searcher consumes energy only when it is actually moving in the search process, and that no energy is being consumed by an individual searcher when it has been disabled or when the source is waiting to reinitiate the sending out of the searcher.

Let $J(N)$ be the lower bound estimate to the amount of energy expended in the search that is obtained by assuming that as soon as any one of the searchers has found the object, then all the other searchers will also stop their search, and that energy is only expended during the movement of the searchers in proportion to the time spent in searching. Thus $J(N)$ is proportional to N times the expected effective travel time $E[\tau_{eff}]$ of each of the N searchers, where

$$E[\tau_{eff}] = (1+E[T^*])\int_0^\infty f_i(z_i)dz_i$$

and $J(N)=N.E[\tau_{eff}]$. From the previous analysis we obtain

$$J(N) = [e^{-2D(\lambda+r+a/b-\sqrt{b^2+2c(\lambda+r+a)})} - 1]\frac{1}{\lambda+r+a}. \quad (28)$$

In Fig. 8 we see that $J(N)$ is not significantly affected by N for very different values of λ.

In Fig. 9, we vary $J(N)$ against the average time-out $1/r$ for three different values of N, with a very high value of loss rate $\lambda=0.2$ and "perfect ignorance" $b=0$ during the search: we see that for larger N, $J(N)$ is not sensitive to changes in $1/r$. Figure 10 uses a lower loss rate $\lambda=0.01$ so that each search is now much faster. Here we do see that when the number of searchers increases, energy consumption becomes less sensitive to $1/r$. Figure 11 on the other hand shows that the number of searchers N has little effect on $J(N)$ when the loss rate is very high $\lambda=0.2$ and $1/r$ varies across a large range of values.

In order to see how N should we chosen to optimize both delay and energy consumption, we have plotted the locus of $J(N)$ and $E[T^*]$ when $1/r$ is varied for $b=0.2$ and $\lambda=0.01$, a

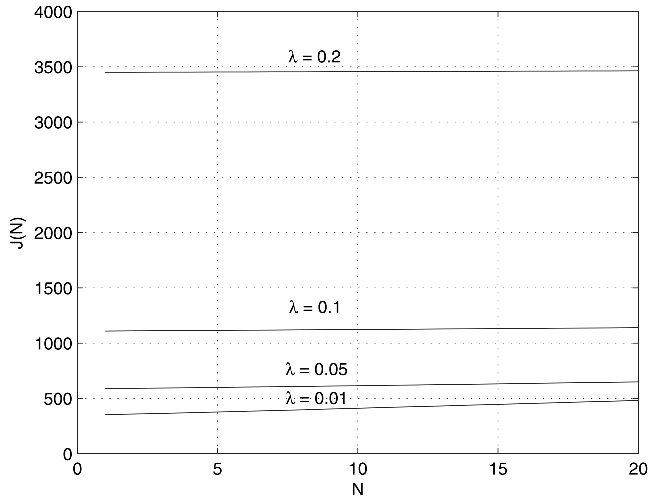

FIG. 8. Lower bound to the expected energy consumption versus the number of searchers. The parameters are $b=0$, $c=1$, $\mu=0.05$, $D=10$ for different values of λ.

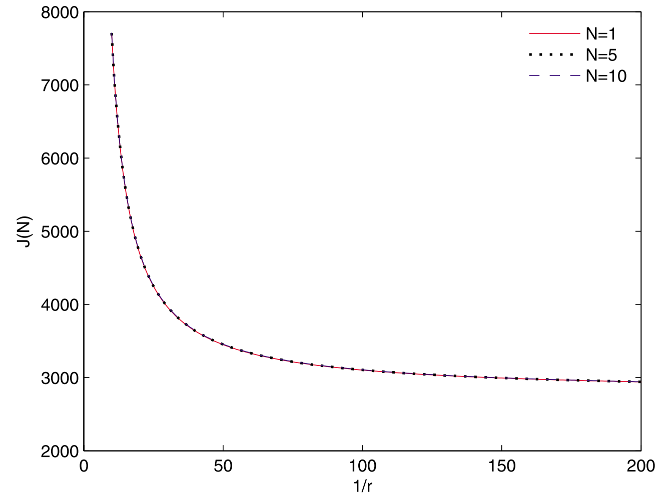

FIG. 10. (Color online) The average energy consumption $J(N)$ versus the average time-out $1/r$ with $D=10$, $b=0$, $c=1$, $\mu=0.05$, and $\lambda=0.2$.

favorable condition, in Fig. 12, and the unfavorable case where $b=0$ and $\lambda=0.15$ in Fig. 13. WE see in both cases that it should be possible to find an operating point with an appropriate value of $1/r$ where both energy and close to minimum.

IV. CONCLUSIONS

This paper presents a model for search by N agents in an unbounded random environment. We assume that a time out is used to eliminate searchers which have searched too long without yielding a result, and that when this happens a new searcher is launched to replace the one that has been removed; all searchers behave independently of each other but with identical statistical behavior.

We derive an expression for the time it takes to find the object being sought as a function of the distance from the source to the object, using a multidimensional Brownian process. The model allows for the loss or destruction of searchers and their finite lifetime, and it includes parameters which characterize the randomness of the search process. As long as a new searcher is sent out to replace one that died or got lost, and that the search process is random so that previous mistakes are not systematically repeated, we show that the object being sought will be found in a finite time if the distance to the object from the source is finite. Depending on the parameters of the system being considered, N can either favorably or adversely affect the average search time. Similarly, the average value of the time out has a very significant impact on the search time, and its can be used to optimize both the search time and the energy being consumed. We therefore also develop estimates of the energy consumed. In the case when the distance from the source to the object is a random variable, we have also shown in the case where $N=1$ that the average time it takes to find the object may be finite or infi-

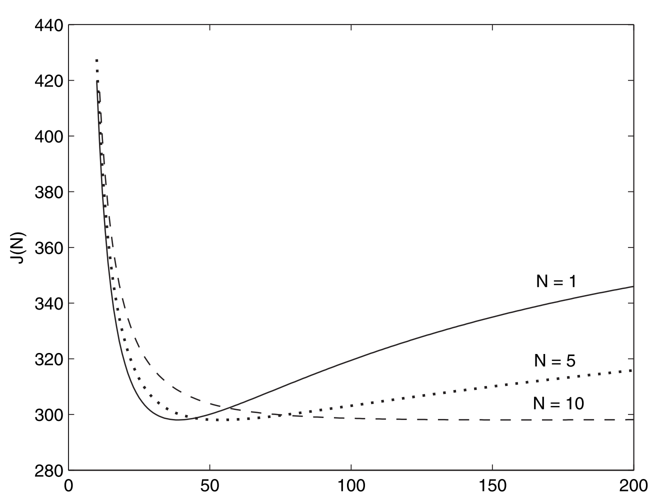

FIG. 9. The average energy consumption $J(N)$ versus the average time out $1/r$ with $b=0.1$, $c=2$, $\lambda=0.01$, $\mu=0.05$, and $D=10$.

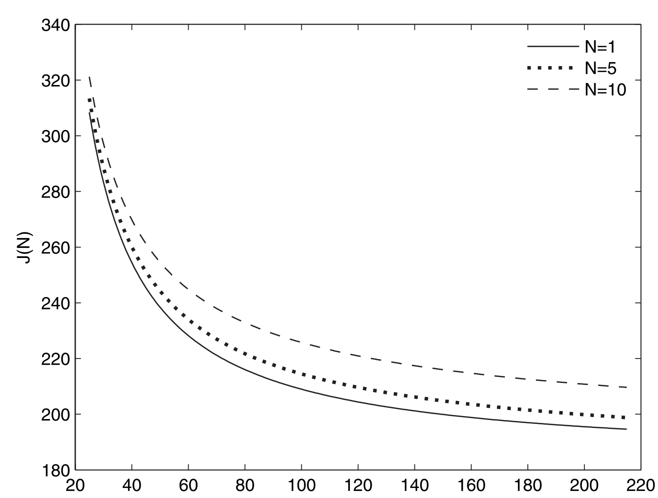

FIG. 11. The average energy consumption $J(N)$ versus the average time-out $1/r$ with $D=10$, $b=-0.1$, $c=1$, $\mu=0.01$, and $\lambda=0.05$.

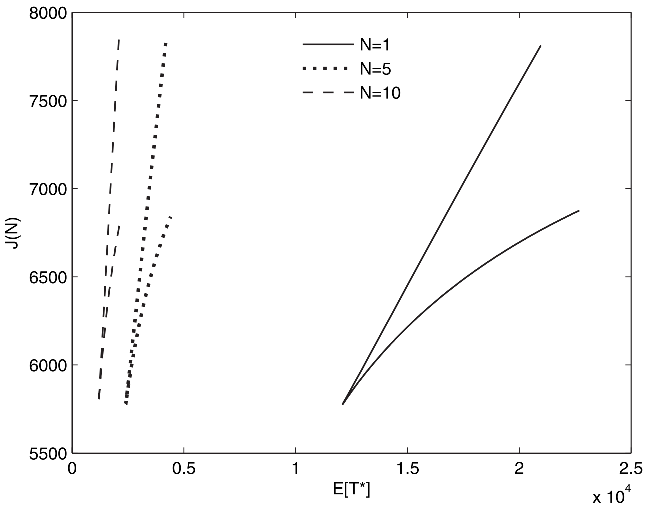

FIG. 12. The locus of the average effective packet delay $E[T^*]$ and the energy consumption $J(N)$ when the average time-out $1/r$ is varied. The parameters are $D=10$, $b=0.2$, $c=1$, $\mu=0.05$, and $\lambda=0.01$. For these low packet loss rates, the minimum energy consumption is obtained when the average travel time is also with minimum.

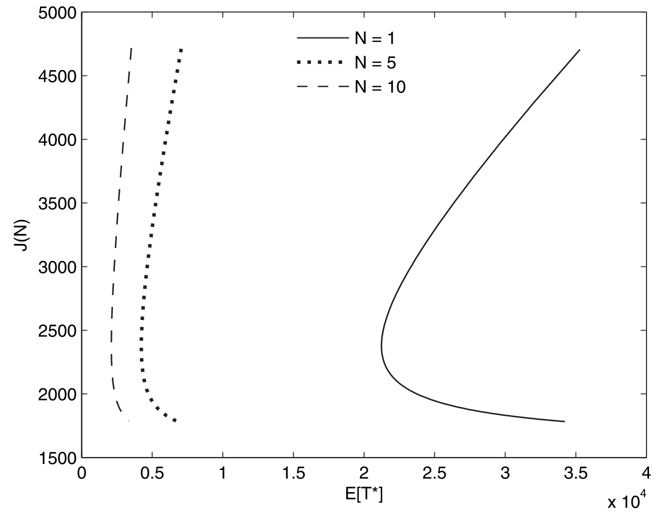

FIG. 13. The locus of the average effective packet delay $E[T^*]$ and the energy consumption $J(N)$ when the average time-out $1/r$ is varied. The parameters are $D=10$, $b=0$, $c=1$, $\mu=0.05$, and $\lambda=0.15$. For such high packet loss rates, the minimum effective travel time does not coincide with minimum energy consumption.

nite, depending on the probability distribution of the distance.

We have presented several numerical examples to illustrate our results and notice that it should be possible to minimize the average search time and the average energy consumption by an appropriate choice of the time out.

From this work, many interesting problems and extensions can arise. For instance, it would be useful to develop models for the case where the object being sought is moving or even escaping from the searchers. Also, the approach that we have developed raises the issue of how communication or learning among the searchers may improve the search. Another interesting question arises when we assume that we are willing to accept to stop the search when we find an object which is approximately similar to the object sought.

One could also consider the case when there are in fact more than one, or even an infinite number of objects which are similar to the one being sought, and we could then study the time it takes to find the first M objects, or we could determine the rate at which objects are found. Yet another challenging question arises when some of the objects being sought, such as explosive mines [18], may actually destroy some of the searchers. Thus we feel that this paper formulates and solves a particular problem, but that it raises a large class of other problems and offers a possible method for addressing a class of research issues.

[1] C. E. Perkins, *Ad Hoc Networking* (Addison Wesley, New York, 2000).
[2] E. Gelenbe, Commun. ACM **52**, 66 (2009).
[3] A. Mademlis, P. Daras, D. Tzovaras, and M. G. Strintzis, Pattern Recognit. **42**, 2447 (2009).
[4] C. Ribrault, A. Triller, and K. Sekimoto, Phys. Rev. E **75**, 021112 (2007).
[5] Y.-B. Ko and N. H. Vaidya, in *Proceedings of the 4th Annual International Conference on Mobile Computing and Networks ACM MOBICOM 1998* (ACM, New York, 1998), pp. 66–75.
[6] E. Gelenbe, in *Proceedings of the Second Workshop on Spatial Stochastic Models for Wireless Networks (SPASWIN'06)* (IEEE, Piscataway, NJ, 2006), pp. 1–6.
[7] E. Gelenbe, ACM Trans. on Sensor Networks **3**, 111 (2007).
[8] B. Tilch, F. Schweitzer, and W. Ebeling, Physica A **273**, 294 (1999).
[9] W. Ebeling, F. Schweitzer, and B. Tilch, Biosystems **49**, 17 (1999).
[10] V. Tejedor, O. Benichou, and R. Voituriez, Phys. Rev. E **80**, 065104(R) (2009).
[11] G. Oshanin, O. Vasilyev, P. L. Krapivsky, and J. Klafter, Proc. Natl. Acad. Sci. U.S.A. **106**, 13696 (2009).
[12] F. Rojo, J. Revelli, C. E. Budd, H. S. Wio, G. Oshanin, and K. Lindenberg, J. Phys. A: Math. Theor. **43**, 345001 (2010).
[13] A. Einstein, *Investigations on the Theory of Brownian Motion* (Dutton, Dover, New York, 1926).
[14] E. Gelenbe, J. ACM **22**, 261 (1975).
[15] E. Gelenbe, Acta Inf. **12**, 285 (1979).
[16] J. Medhi, *Stochastic Models in Queueing Theory* (Academic Press, New York, 1991).
[17] E. Gelenbe, X. Mang, and R. Onvural, Perform. Eval. **27-28**, 411 (1996).
[18] E. Gelenbe and Y. Cao, Eur. J. Oper. Res. **108**, 319 (1998).